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Dominant spanwise Fourier modes, and the existence of very large scale coherence in turbulent boundary layers

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Abstract

Multiple plane stereo PIV results and data from a rake of ten hot-wire probes are used to investigate the largest scale structures in a zero-pressure-gradient turbulent boundary layer. Instantaneous vector fields from stereo PIV in spanwise-streamwise planes reveal long low- and high-speed regions, with a length that often exceeds the viewing window ($> 2\delta$). Also evident is a remarkable degree of spanwise organisation, that manifests as a persistent spanwise stripiness in the u component of the PIV vector field. Almost all trace of such spanwise organisation is lost in the mean statistics, presumably due to the multitude of scales naturally present in wall-bounded turbulence. This can be overcome by 'de-jittering' the instantaneous vector fields. By sorting the data according to dominant spanwise fourier modes, and then applying simple statistical tools to the sorted subsets, we are able to extract a clear view of spanwise organisation. Results are confirmed in the various PIV data-sets. Since the PIV fails to adequately capture the full streamwise extent of the low-speed regions, a rake of hot-wire probes is also employed to capture a continuous view of the spanwise coherence. It is found that the low-speed regions are in fact extremely persistent in the streamwise direction, often exceeding 20 δ in length. The fact that these long features meander appreciably in the spanwise direction will limit the overall streamwise length-scale as witnessed by a single probe or single point statistic. For instance, premultiplied one-dimensional spectra of the streamwise velocity ($k_x\Phi_{uu}$) at this z/δ show a peak contribution for characteristic lengthscales of $5-7\delta$.

Introduction

It is well documented that a streaky structure with an average spanwise streak spacing of approximately 100 wall units exists in the near-wall region of turbulent boundary layers (See [8] for review). Further from the wall, two-point correlations obtained from hot-wire data ([10, 7]) have consistently indicated a larger spanwise structure in the log and wake regions (scaling on δ and increasing in size with distance from the wall). Prior to the advent of PIV, the precise form of these log-region structures was largely unproven, although statistics based on fluctuating u signals at these heights (particularly the peak in the pre-multiplied energy spectra $k_x\Phi_{uu}$ and the long tails in the autocorrelations) had long hinted at the existence of long regions of uniform streamwise momentum. PIV measurements in the streamwise-spanwise plane revealed that the log region is indeed charcterised by it's own streaky structure (eg [9]), albeit of a much larger scale. Long regions of streamwise momentum deficit are found, with high-speed fluid seeming to fill the separation between neighbouring motions. Further PIV investigations have suggested that these long modes of uniform momentum deficit are associated with packets of hairpin vortices [1, 3]. The low speed regions are found to be of the order $0.4-0.5\delta$ wide, and typically have a length that exceeds the streamwise extent of the PIV frame (usually limited to $\sim 2\delta$). More recent analysis of large numerical domain DNS results (in particular 2D spectra) have shown that Φ_{uu} energy can reside in very long streamwise modes (certainly $> 20\delta$) for larger k_y bands [2].

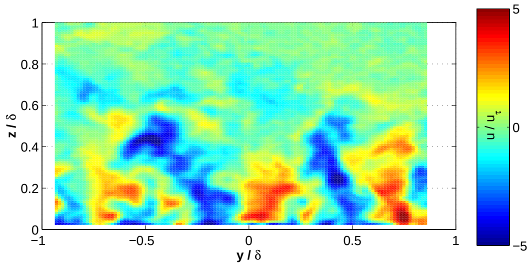

Figure 1: Example streamwise velocity field from 45° inclined plane. Color shading denotes velocity fluctuation.

Facility

We investigate these scales using three separate datasets, all of which were obtained at the same Reynolds number ($Re_\tau = 1100$) and in the same flow facility (open return suction-type boundary layer wind-tunnel of working section $4.7\times1.2\times0.3$ m). The datasets comprise:

i. Inclined plane cross-stream PIV measurements taken at both 45° and 135° to the x-axis.
ii. Streamwise-spanwise plane PIV measurements.
iii. Hot-wire measurements from a rake of 10 single wire sensors covering approximately 1.14δ in the spanwise direction.

The axis system x, y and z refer to the streamwise, spanwise and wall-normal directions, with u, v and w describing the respective velocity components. Dataset (i) is described in full detail in [5]. Dataset (ii) is described in [3]. Combined correlation results from (i) and (ii) are presented in [4].

Interpreting two-point correlation R_{uu}

Figure 1 is an example streamwise velocity field from the 45° inclined plane. This frame is typical of the many hundreds of captures. Large eruptions of low-speed fluid (marked in blue) extend from the wall, often growing beyond $z = 0.5\delta$. Such regions are flanked in the spanwise direction by high-speed fluid (marked in red). There are also signs of weaker high-speed regions crowning the low-speed motions. Such arrangements are typical of inclined hairpin type structures (discussed in [5]). In Figure 1 the complete pattern repeats in the spanwise direction, giving a strong impression of spanwise periodicity. Such repetition is evident in many of the acquired vector fields.

Figure 2 shows the two-point correlation based on the streamwise velocity fluctuations (R_{uu}) about the reference point $z_{ref}/\delta = 0.14$, for the same 45° inclined plane. The picture we see is familiar. A central region of positive correlation with a spanwise width of approximately 0.5δ is flanked by anti-correlated lobes, separated by approximately 0.75δ. These two length-scales seem to approximately correspond to the width and separation between the low- and high-speed regions

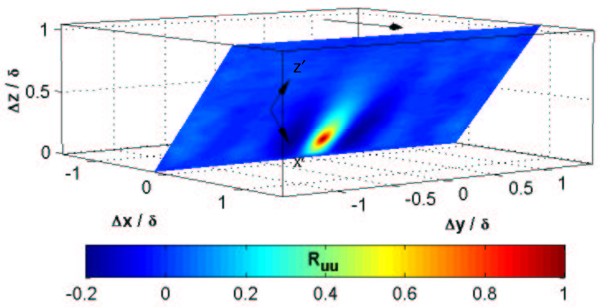

Figure 2: R_{uu} at $z_{ref} = 0.14\delta$ for the 45° inclined plane data.

shown in Figure 1. If we calculate the spanwise energy in the streamwise velocity signal, a peak in the pre-multiplied spectra ($k_y \Phi_{uu}$) is noted at a spanwise length-scale of approximately 0.75δ. It is incorrect at this stage to interpret this peak as signifying a true spanwise periodicity. Indeed it can be shown that a single low-speed region of width 0.5δ, flanked by similar sized high-speed regions could lead to such an energy peak. In other words, the velocity signature due to a single hairpin or hairpin packet, in the absence of any spanwise repetition of this pattern, could produce a similar two-point correlation and peak in the spectra. So the question we asked is *why doesn't the spanwise repetition so obvious in the instantaneous flow-fields manifest in the two point correlation as a spanwise ringing of the positive correlation region?* The reason of course lies in the multitude of scales that reside in turbulent flows. This causes a statistical smearing that masks any signs of repetition. To overcome this, a very simple method of sorting is proposed.

De-jittered data

The method is as follows:

- A spanwise trace of streamwise velocity fluctuation is extracted from each frame at a given reference height (in this case $z_{ref} = 0.14\delta$).
- Fourier analysis of the extracted signal reveals the dominant spanwise mode in that particular frame.
- The frame is sorted or 'binned' according to the dominant mode (λ_y).
- Two point correlations are conducted on the 'binned' (or de-jittered) sets of frames.

The bin sizes used are relatively broad (0.25δ increments from 0 to 1.5). It is found that most of the energy resides in the first 4 modes, with $0.5 < \lambda_y/\delta < 0.75$ being the most populated bin. Here λ_y is spanwise wavelength. Of the entire data-set, 37% of all frames exhibit a dominant spanwise u fluctuation of this wavelength. The two-point correlations as calculated on these first four modes are shown in Figure 3 plots (a - d). Note that these modes recover 93% of the total energy (93% of all PIV frames exhibit these spacing modes). With the data sorted in this way, there is clear evidence of ringing in the 'binned' two-point correlations indicating an underlying spanwise periodicity. This is perhaps expected along the line $z = z_{ref}$, however it is noted that these spanwise modes extend a considerable distance in the wall-normal direction ($> 0.5\delta$). If we take the bin $0.5 < \lambda_y/\delta < 0.75$ (plot b), the implication is that 37% of the PIV images have a large-scale spanwise periodicity, of repeating high- and low- momentum regions, extending a considerable distance across the boundary layer. Note that this is raw unfiltered data. Yet 93% of all frames are well described by a

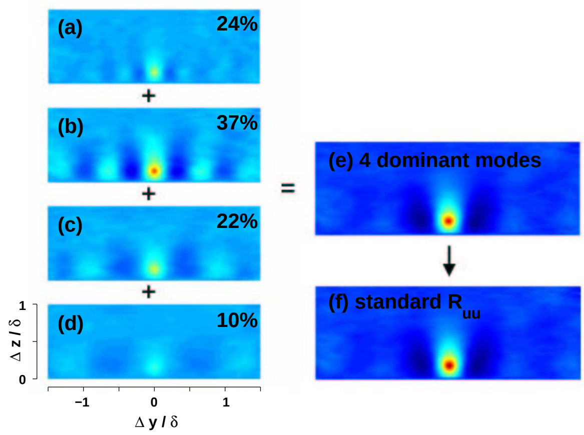

Figure 3: R_{uu} calculated at $z_{ref} = 0.14\delta$ for each of the four dominant modes (a) $0.25 < \lambda_y/\delta < 0.5$; (b) $0.5 < \lambda_y/\delta < 0.75$; (c) $0.75 < \lambda_y/\delta < 1.0$; (d) $1.0 < \lambda_y/\delta < 1.25$. (e) shows the sum of these four modes as compared to (f) the standard R_{uu}.

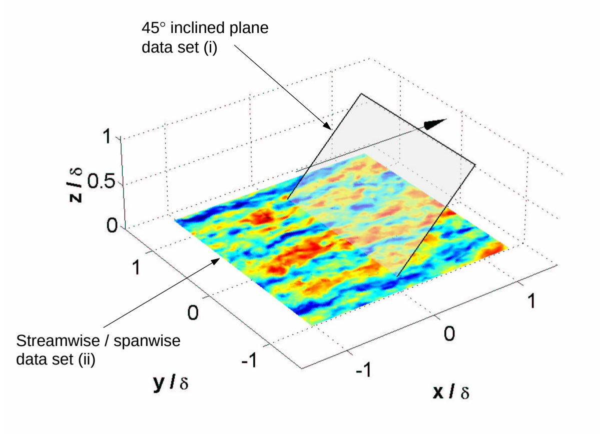

Figure 4: Example u velocity fluctuations from streamwise / spanwise plane PIV. Color shading as Figure 1.

single sinusoidal mode for all of the log region and slightly beyond (up to 0.5δ). It is worth noting that despite the fact that repeating modes characterise the majority of the data, the superposition of the individual modes leads to an R_{uu} plot that exhibits no obvious sign of spanwise periodicity. Therefore, care must be taken when interpreting statistical quantities, such as R_{uu}, where multiple scales interactions can mask underlying organization. As a final clarification of this, Figure 3(e) shows the sum of the 4 dominant modes which almost completely recovers the standard R_{uu} profile included as plot (f).

Spanwise-streamwise plane PIV data

The view of this large-scale spanwise periodicity can be further enhanced by applying the same de-jittering technique to dataset (ii). Figure 4 is an example of the u velocity fluctuation in the streamwise / spanwise plane. The spanwise stripiness observed by [9, 3] is evident. The data set is similarly 'binned' according to the dominant spanwise mode at the centre of the frame ($x/\delta = 0$). Since data set (ii) was obtained at $z/\delta = 0.14$, the resulting R_{uu} results can be matched with the previous plots shown in Figure 3 (where $z_{ref}/\delta = 0.14$), with the planes intersecting at the condition point $x/\delta = 0$. Figure 5 shows such a construct for the most populated bin ($0.5 < \lambda_y/\delta < 0.75$).

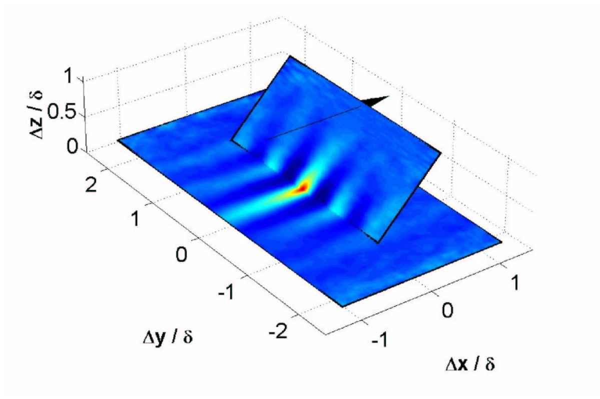

Figure 5: R_{uu} calculated from datasets (i) & (ii) at $z_{ref} = 0.14\delta$ for the mode $0.5 < \lambda_y/\delta < 0.75$

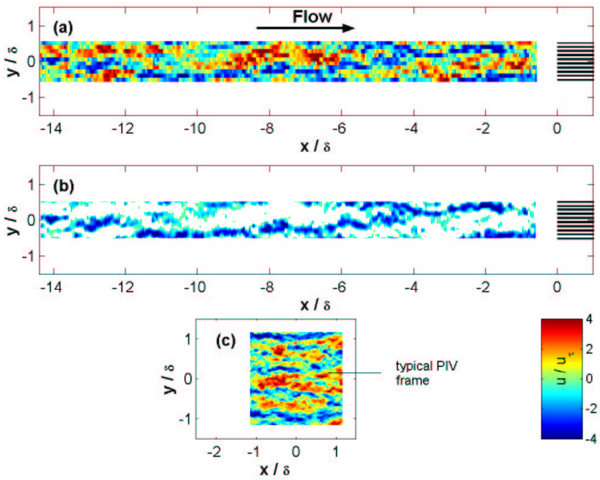

Figure 6: (a) Example signal section from ten sensor hot-wire rake at $z/\delta = 0.14$. Spatial view is reconstructed using local mean velocity ($x = -U_c t$); (b) the negative momentum fluctuations only; (c) comparison with typical PIV frame.

Clearly the previously observed spanwise modes actually persist for a substantial distance in the streamwise direction, extending well beyond the $x = \pm\delta$ view afforded by the PIV data.

Hot-wire data

A spanwise rake of ten single sensor hot-wire probes was inserted into the boundary layer at a height from the wall of $z/\delta \approx 0.14$. The ten sensors were separated by 0.114δ in the spanwise direction, such that the entire rake measured a spanwise domain just greater than one boundary layer thickness. The idea here is to use the fluctuating signals from the ten probes to reconstruct the instantaneous spanwise profile of the u velocity fluctuation. By projecting this signal in time and using Taylors Hypothesis (frozen convection) a view of the long high- and low-speed regions can be constructed that covers a much larger streamwise domain than that available with PIV. An example section of the reconstructed field is shown in Figure 6(a). A typical size PIV vector field for this Reynolds number is also shown as plot (c). Clearly there are some very long features in the flow that the PIV data will fail to adequately capture. Figure 6(b) highlights the streamwise extent of the low-speed region by shading only those regions of negative velocity fluctuation. A long, meandering low-speed region wanders through the measurement domain for the entire 14δ shown. Indeed when we run movies of the frozen turbulence as it advects past the probe array, there are many instances where the length of the low-speed regions exceed 20δ. There is a problem that the meandering often causes these regions to leave the spanwise limit of the domain ($y/\delta = \pm 0.5$). This tends to curtail the maximum length of low-speed regions that we can track. To overcome this problem, and track the full extent of the very longest meandering features it would be useful to increase the spanwise domain measured by the rake to at least 2δ. At present we have no concrete statistics on the probability distribution of these large events. However, in the future, it is hoped that streak tracking algorithms can be developed to successfully identify these regions.

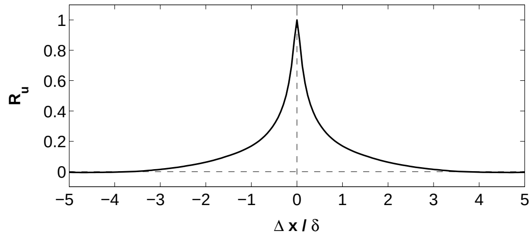

Figure 7: Autocorrelation curve from the fluctuating hot-wire u velocity signals. The spatial x-ordinate is obtained using Taylors hypothesis and a convection velocity based on the local mean $\overline{U} \approx 0.64 U_\infty$.

Figure 7 shows the autocorrelation for all ten probes. Note that the positive correlation region tends to fall to zero (becoming negative) for signal shifts $\Delta x > 4\delta$. Similarly the broad peak in the pre-multiplied streamwise spectra $k_x \Phi_{uu}$ occurs for comparable length-scales. Such features in classical single point statistics have previously informed our view of the largest energetic scales in turbulent boundary layers. However the rake data, and in particular velocity maps such as those shown in Figure 6 (a & b), demonstrate that much larger scales inhabit the flow. It is proposed that these length-scales are not resolved by classical single point techniques due to a spanwise wandering or meandering in these structures. In fact there is a clue to such behaviour in the autocorrelation result shown in Figure 7, where there are signs of anti-correlated regions beyond signal shifts of approximately $\pm 4\delta$. Figure 8 clarifies this behaviour. A simple model of a long meandering low-speed region is shown, flanked on either side by high-speed regions. A region width of 0.5δ is chosen, with meandering amplitude and wavelength of 0.6δ and 16δ respectively. At this stage the precise form is arbitrary. However Figure 8(a) confirms that a long meandering feature would be witnessed by a single-point measurement as a much shorter event. Figure 8(b) shows the autocorrelation calculated on a randomly spaced array of such features (with a normal distribution of lengths about 0 to a maximum 40δ). Despite the existence of what is essentially a very large scale feature, the positive correlation region seems to drop to zero (becoming negative) at a signal shift of $\pm 3\delta$. This represents something like the average streamwise path-length detectable through the low-speed event by a single-point measurement.

There is additional supporting evidence for these large meandering regions contained within the two-point correlation results. Figure 9(a) shows the R_{uu} map obtained from the hot-wire rake. Figure 9(b) shows the corresponding R_{uu} result from the streamwise-spanwise PIV data. Good general agreement is seen between the two, although the limited field of view of the PIV data is again evident. Even with the extended view afforded by the hot-wire data, the central positive correlation region drops to zero relatively quickly. This is previously predicted by the

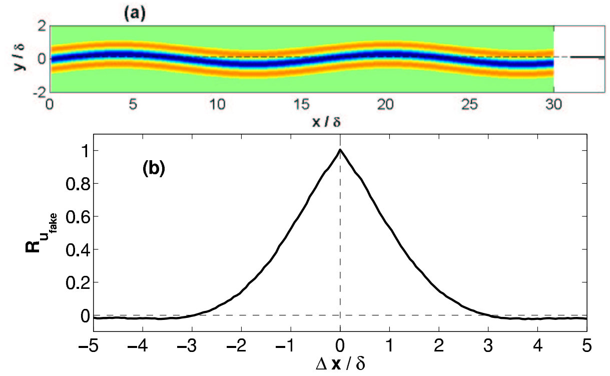

Figure 8: (a) Fake meandering high- and low-speed regions; (b) Autocorrelation associated with random distribution of such regions of varying lengths.

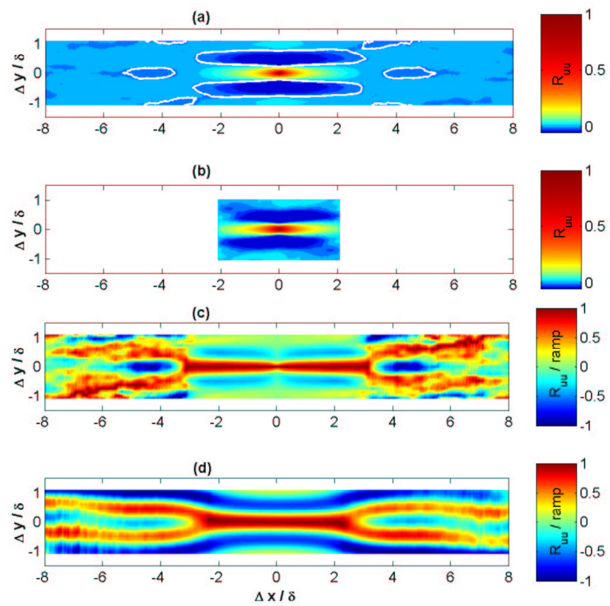

Figure 9: (a) Two-point correlation from hot-wire data (contours show $R_{uu} = -0.002$); (b) same result from streamwise / spanwise PIV; (c) hot-wire correlation normalised by the ramp function $(\max|R_{uu}(\Delta x)|)$; (d) fake streak correlation normalised by ramp function.

autocorrelation results of Figure 7. However, there are weakly correlated regions forming a distinctive **X** shape across the *Ruu* map. Such features could not be resolved from the limited field PIV. These are difficult to see with the colour axis as shown in plot (a). However, we can highlight these features by normalising the R_{uu} map with the modulus of the maximum correlation value for any given spatial shift Δx. Normalising by this ramp function overcomes the problem of the rapid drop-off in absolute correlation magnitude as Δx is increased from the reference point. Plot (c) clearly exhibits some weak yet significant correlation behaviour in the far field, that could be due to a meandering motion in the largest scales. Figure 7(d) shows a similarly normalised two-point correlation result for the random array of idealised fake low-speed regions as introduced in Figure 8(a). The similarity between plots (c) and (d) would imply that the meandering streak is a reasonable model for these large-scale structures. We can state that a random array of meandering large-scale features can cause a similar two-point correlation result to that obtained from the hot-wire data.

Conclusions

An analysis of the largest scale features in the log region of a turbulent boundary layer has lead to the following conclusions:

- Instantaneous spanwise *u* behaviour is well described by single spanwise sinusoidal modes extending a considerable distance in the wall-normal and streamwise directions
- There is a strong spanwise periodicity associated with the largest streamwise velocity fluctuations. Since, these velocity fluctuations are indicative of a wider vortical structure, such results would seem to have implications to flow control / prediction strategies, and may hold clues to the underlying structural dynamics.
- These large-scale features are extremely long in the streamwise direction (occasionally $> 20\delta$) and seem to meander appreciably. The meandering effectively hides the true length of these features from single point measurement techniques.
- Such large-scale motions have previously been observed in pipe flows from single point measurements [6]. This raises the interesting possibility that the radial nature of pipe boundary layers acts to restrict spanwise meandering.

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