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Evidence of the -1-law in a high Reynolds number turbulent boundary layer

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Abstract

Dimensional analysis leads to a prediction of a -1-power-law for the streamwise velocity spectrum in a turbulent boundary layer. This law can be derived from overlap arguments or from physical arguments based on the attached eddy hypothesis of Townsend (1976). Some recent experiments have questioned the existence of this power-law region in wall-bounded flows. In this paper experimental spectra are presented which support the existence of the -1-law in a high Reynolds number boundary layer, measured in the large boundary layer facility in the Walter Basset laboratory at the University of Melbourne. The paper presents the experimental results and discusses the theoretical and experimental issues involved in examining the existence of the -1-law and reasons why it has proved so elusive.

Introduction

Turbulent wall-bounded flows occur in a wide range of situations that are of interest both for technical, engineering applications and in terms of the general physics of turbulent flows. The practical interest stems from the fact that many flows of engineering importance occur at high Reynolds numbers. Examples are flows over submarines, aircraft and the atmospheric surface layer. The theoretical interest stems from questions regarding the asymptotic behaviour of such flows as the Reynolds number tends to infinity. At present many experiments and direct numerical simulations are limited to lower Reynolds numbers and the important question is how to extrapolate these results to higher Reynolds number flows. There are a few theoretical predictions concerning the likely characteristics of these flows at high Reynolds number which have been used in modelling and predicting real flows. One of these is the -1-power-law for the Power Spectral Density (PSD) of the streamwise fluctuating velocity component. This law can be derived either by simple dimensional reasoning or, more rigorously from asymptotic overlap arguments as described, for example, in Perry *et al.*(1986). These arguments are consistent with the attached eddy hypothesis of Townsend (1976). Essentially, the idea is that there is a range in wavenumber space in which the effects of both viscosity and the outer length-scale (eg. the boundary layer thickness or the pipe radius) are negligible. These arguments are discussed in more detail in the next section. The salient feature is that an overlap region exists where both *inner scaling* and *outer scaling* are simultaneously valid. Inner scaling refers to non-dimensionalisation using the length scale z, the wall-normal position, and the velocity scale U_τ, the friction velocity. Outer scaling uses the length scale δ, the boundary layer thickness, and the velocity scale U_τ. Despite the fact that the arguments involved in deriving such a region are, at least, very plausible, the experimental evidence for the -1-law has not been completely convincing. This has led to questions regarding the possible existence of such a region. Recently, Morrison *et al.* (2004) have analysed spectra taken in the Princeton *superpipe* and concluded that for those data no overlap region is observed and therefore no universal -1-law is present. They refer to this as 'incomplete similarity". Del Alamo *et al.* (2004) interpret the incomplete similarity result as a consequence of a variation of velocity scale between the largest eddies in the flow (scaling with δ) and the eddies that scale with z, the local distance from the wall. Consequently they show that a logarithmic correction is needed to the -1 law. All of the above arguments agree that the existence of a -1 law depends on a sufficiently large Reynolds number to ensure an overlap region. What remains unclear at this stage is what value of Reynolds number qualifies as sufficiently large, and where measurements should be made in the boundary layer in order to discover such a region. In what follows we present new measurements in the high Reynolds number boundary layer at the University of Melbourne both at high Reynolds number close to the wall. The measurements show evidence of a -1-law region with complete similarity.

Experimental Details

The wind-tunnel facility

The measurements were conducted in the high Reynolds number boundary layer facility in the Walter Basset Aerodynamics Laboratory at the University of Melbourne as shown in figure 1. This unique facility is especially designed for the experimental study of high Reynolds number boundary layers with sufficient thickness to provide for good spatial resolution. The facility is a blower-tunnel driven by a 2m diameter axial flow fan. Air from the fan passes through two vaned corners, through a settling chamber with honeycomb and screens, then through a 6:1 contraction and into the working section. The working section is the unique feature of the tunnel. It has a cross-section of 2m x 1m (horizontal x vertical) and 27m in length. The long working section allows the boundary layer to grow over a long fetch and hence produces a high Reynolds number, thick, boundary layer. The boundary layer near the end of the working section is approximately 330mm thick, providing for good spatial resolution of the measurements. Separate experiments in another wind-tunnel (Jones *et al.* 1995) show that the effect of finite width is negligible as long as the tunnel is wider than six boundary layer thicknesses (6δ). The tunnel is capable of a maximum speed of 45 m/s, although the measurements to be presented here are for lower tunnel speeds. At a speed of 30m/s the free-stream turbulence intensity is 0.05%. The working section is above atmospheric pressure and a zero pressure-gradient is achieved by bleeding air from the tunnel ceiling through adjustable slots. The coefficient of pressure, C_p, along the working section is constant to within $\pm 1\%$.

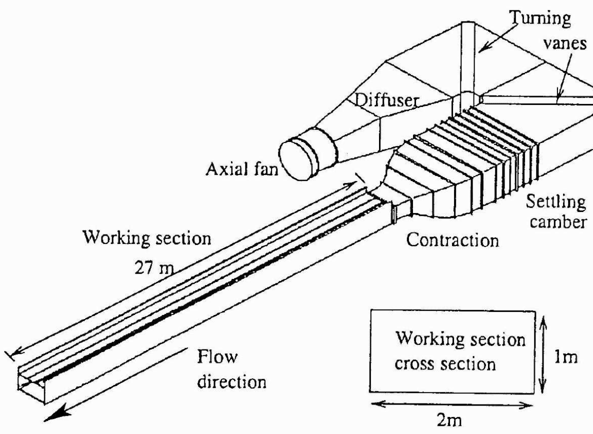

Figure 1: Schematic of wind-tunnel used in this study.

Hot-wire measurements

The measurements to be presented here have been made using hot-wire probes made from platinum Wollaston wires. All measurements shown were made with wires of 2.5μm diameter etched to 0.4mm in length (giving a length to diameter ratio, l/d=160). The wires were run at constant temperature at an overheat ratio of 1.8 using an AA-labs model AN1003 anemometer. The system was checked to ensure second order impulse response and showed a frequency response of greater than 150kHz. The wires were calibrated statically by traversing them into the centre of the tunnel in the undisturbed free-stream. They were calibrated using a third-order polynomial fit to the discrete data. The signal was filtered at 35kHz using 8th order analogue Butterworth filters, to avoid aliasing, and were sampled at 80kHz using a 16 bit A-D converter board mounted in a PC. The probes were attached to a stepper motor driven traverse run by the computer and could be positioned within the boundary layer with an accuracy of 3μm. The initial wall distance was measured using a video camera with a short focal length lens to image the probe and its image in the polished aluminium floor of the tunnel. The initial position could be determined to an accuracy of $\pm 5\mu$m.

The PSD was determined by sampling bursts (or "records") of the velocity signal of 3.27 seconds in length at 80kHz (262,144 points). This was determined to be sufficiently long to capture the lowest wavenumber motions with significant energy content. Each burst is then fast-Fourier-transformed online and the PSD formed from the modulus squared of the transformed signal. In order to ensure convergence of the PSD a series of 500 bursts were taken and the PSDs were ensemble averaged to form the final PSD. Five hundred bursts corresponded with approximately 45 minutes total time to measure a single spectral curve (for a single wall-normal position). Extreme care was taken to ensure accurate and well-converged data for each measurement. One typical wall-normal position of interest was averaged over 5000 bursts (approximately 7.5 hours) and compared with the 500 burst case to ensure that the results were equivalent in all essential details. Calibration drift was checked by re-calibrating at the end of every run and proved to be negligible for all cases (the temperature of the tunnel remained constant to within $\pm 1^{o}C$). In order to further reduce the scatter in the data points of the original high resolution PSD file (131,072 points), the data was resampled by averaging 27 points of the original file to form one point in the final file (at the centre of the 27 point window). This was done with non-overlapping windows in spectral space so that each point in the file was produced from independent data. This is equivalent to the approach of using a narrow band-pass filter to evaluate the PSD from an analogue velocity signal (as used before the advent of digital sampling equipment). The final file had 4800 points. It should be noted that at the very lowest wavenumbers a smaller window was used so as to avoid discarding data at the very low end (this is for appearance of the results to show the full PSD curve). No other form of "smoothing" was used on the data.

Scaling the spectra

As noted earlier the existence of a -1-law, as predicted by the theory, requires, not only a region with a -1-law behaviour, but that this region must scale with both inner and outer flow scaling. Inner flow scaling requires that,

$$\frac{\phi_{11}(k_1 z)}{U_\tau^2} = \frac{\phi_{11}(k_1)}{zU_\tau^2} = f(k_1 z), \tag{1}$$

and outer flow scaling requires that

$$\frac{\phi_{11}(k_1 \delta)}{U_\tau^2} = \frac{\phi_{11}(k_1)}{\delta U_\tau^2} = f(k_1 \delta), \tag{2}$$

where $\phi_{11}(k_1 z)$ is the PSD of the streamwise velocity fluctuation per unit non-dimensional wavenumber $k_1 z$ and z is the wall-normal position. In order to examine this scaling it is necessary to know the mean flow parameters. These were taken from Pitot-tube measurements of the mean velocity profiles. The wall shear stress velocity, U_τ, was found using the method of Clauser which assumes a logarithmic region in the mean velocity profile with universal constants. This technique has been shown to be accurate by comparison with direct measurements using oil-film interferometry, in the same wind-tunnel as used in this study, by Jones *et al.*(2001) who also checked and confirmed that the mean velocity variation was logarithmic. The boundary layer thickness definition used is that of Perry *et al.*(2002) i.e.

$$\delta = \delta^* C_1 / S \tag{3}$$

where δ^* is the displacement thickness of the boundary layer, $S = U_1/U_\tau$ (U_1 is the free-stream velocity) and C_1 is a constant found from integrating the velocity-defect profile i.e.

$$C_1 = \int_0^\infty \frac{U_1 - \overline{U}}{U_\tau} d(z/\delta) \tag{4}$$

The streamwise wavenumber, k_1, was found from the frequency, f, using Taylor's hypothesis of frozen turbulence i.e.

$$k_1 = \frac{2\pi f}{U_c} \tag{5}$$

where U_c is the convection velocity of the turbulent motions past the probe. Different values of U_c have been examined and will be discussed later.

Experimental Results

The results shown here were obtained at a streamwise position 22m downstream from the exit of the contraction tripping device. The boundary layers on all four walls were tripped using grade 40 sandpaper strips located inside the contraction. All profiles were measured with a nominal free-stream velocity of 20m/s (note that for runs on different days the Reynolds number per unit streamwise distance was matched by varying the tunnel velocity to compensate for any change in viscosity due to changes in ambient conditions). The Reynolds number at this point is $R_\theta = 37540$ which corresponds with $\delta_+ = 14380$.

Figure 2 shows the premultiplied spectra in the vicinity of $z^+ = zU_\tau/\nu = 100$ scaled with inner (upper plot) and outer

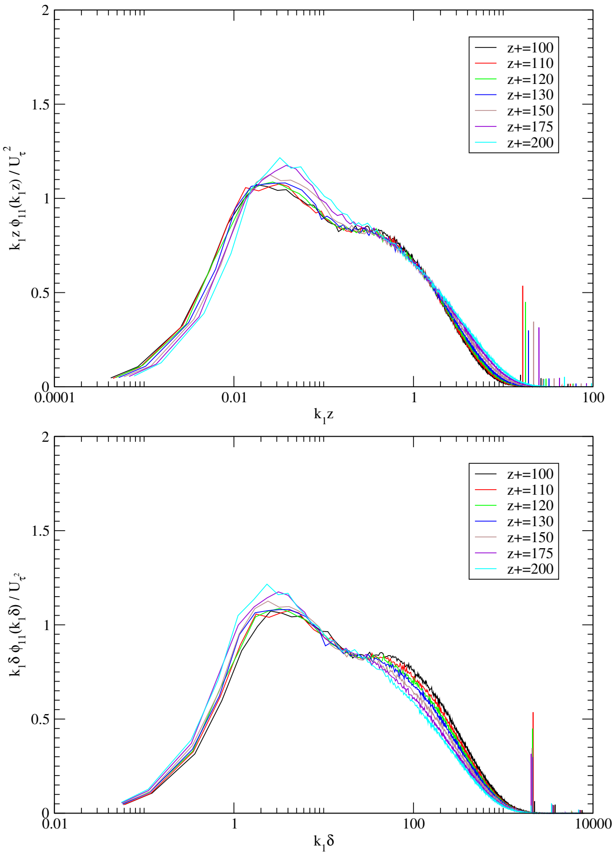

Figure 2: Streamwise spectra in inner (upper plot) and outer (lower plot) scaling. Wavenumber calculated using U_c equal to local mean velocity at each wall-normal position.

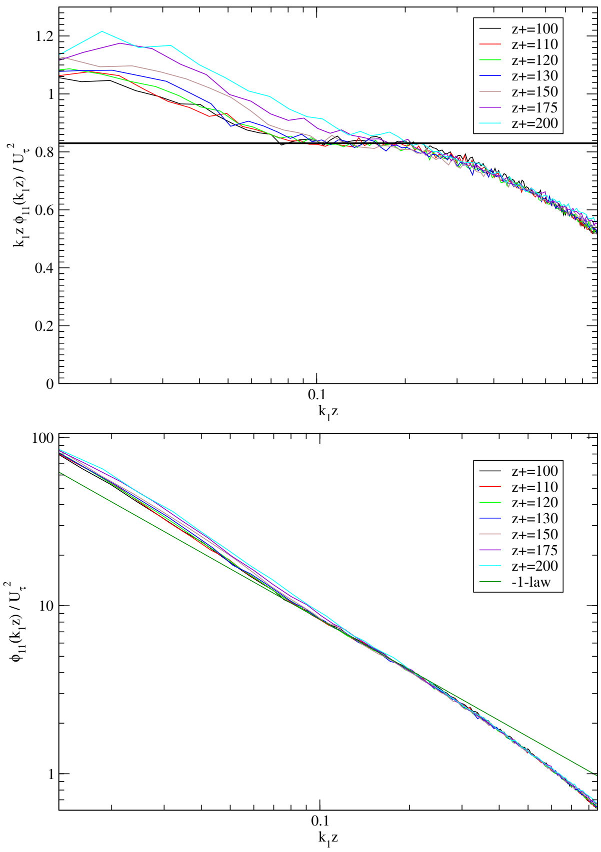

Figure 3: Close up of -1-law region of spectra. Wavenumber calculated using $U_c = \overline{U}$. Upper plot premultiplied. Lower plot standard PSD.

(lower plot) scaling. Here the convection velocity has been taken as the local mean velocity at the wall-normal position of each measurement. This is consistent with standard practice in plotting spectra and hence is shown first: the convection velocity will be discussed further later. It is clear that there is a region where the spectra collapse with both of these scalings. This region corresponds with a plateau in the premultiplied spectra which indicates a -1-power-law behaviour. The region is approximately 1/3 of a decade long in the profile closest to the wall. A close-up of this region is shown in figure 3. The behaviour of the spectra in the two plots is also consistent with the expected scaling behaviour since, in the inner-scaled plot, the profiles collapse at the high end of the region and peel off at the low end as the probe moves away from the wall. This peel off is due to the outer flow scaling of the lower end of the region. In outer-flow scaling the behaviour is also expected. Here the spectra collapse at the low end of the region (albeit not perfectly, see later discussion) and peel off at the high end as the wall-normal distance is increased - this is the effect of changing z. The scaling behaviour of the spectra and the plateau appear to be consistent with complete similiarity arguments and the existence of a -1-law behaviour.

Convection velocity

More information may be gleaned from these plots. The first issue of interest is the correct choice of convection velocity. In figure 2 the local mean velocity was used. It seems plausible that the smaller structures near the wall (and in the vicinity of the probe) may convect at this velocity. Examining the lowest wavenumbers suggests that the collapse with outer flow scaling is good but not perfect. These structures are of a size equal to, or larger than the boundary layer thickness. If these structures extend to the edge of the boundary layer then perhaps a more appropriate convection velocity for the low wavenumbers would be the free-stream velocity, U_1. Figure 4 shows the same spectra with the wavenumber calculated using $U_c = U_1$. The collapse in the low wavenumber range with outer scaling is improved, whereas a very close examination reveals that the collapse of the high wavenumber range with inner-scaling is not quite as good as in figure 2. This suggests the, physically plausible, idea of a spread in convection velocity across wavenumber space. At this stage it is not possible to calculate this spread but it is sufficient to note that it exists. This spread is consistent with the measurements of Krogstad *et al.* (1997) who found a significant spread in convection velocities with the scale of the convected structure. They found that motions of the order of the boundary layer thickness have a $U_c \approx U_1$ whereas the smaller scales convect at a significantly lower velocity. Note that the existence of a spread does not affect conclusions about the existence of a -1-power-law region, in fact a spread in convection velocity of this nature would have the effect of stretching the plateau in wavenumber space and hence increasing its length.

Spatial resolution

The spatial resolution of the measurements is important when evaluating the results. In the measurements presented so far the hot-wire length, l, was 0.4mm which corresponds with $l^+ = 17.5$ at the Reynolds number of the measurements ($\delta^+ = 14380$). Ligrani & Bradshaw (1987) suggest that there is significant attenuation of the streamwise stress if $l^+ > 20 - 25$. There is a way to estimate the point at which spatial resolution limitations start to occur in these measurements. Consider the high wavenumber end of the plots shown in inner-scaling. The different wall-normal positions all collapse though the -1-law

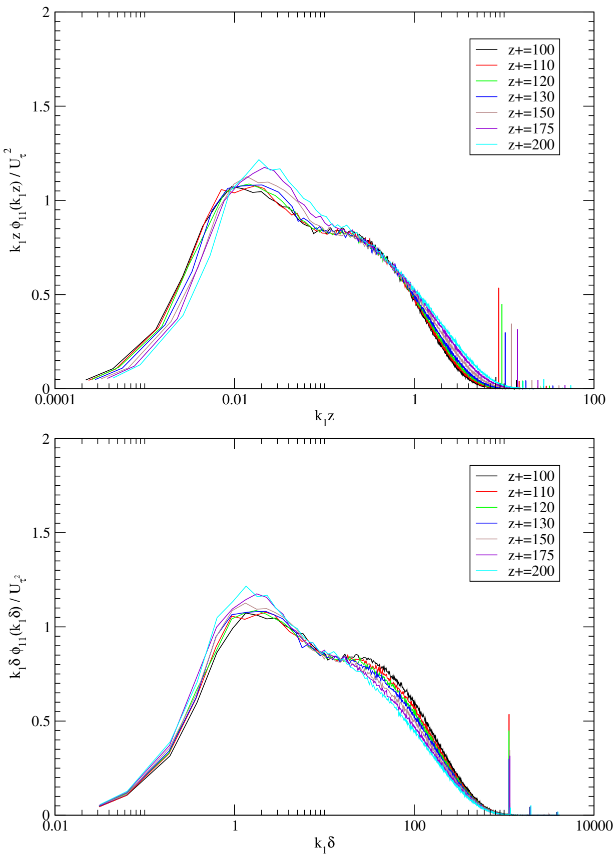

Figure 4: Streamwise spectra in inner (upper plot) and outer (lower plot) scaling. Wavenumber calculated using $U_c = U_1$.

region and beyond, up to a point where they start to peel off. If the spatial resolution was a problem below the peel-off position then the spectra *would not collapse with inner scaling*. This suggests that the -1-power-law region itself is well-resolved.

The limits of the -1-law region

The experiments shown here allow us to make some assessment of the conditions required to obtain a -1-law in the streamwise spectrum. We will use the values of parameters from the plots where $U_c = \overline{U}$ since this is the most common choice and hence will be convenient for other researchers. The outer-flow scaled plot shows that the -1-law starts at approximately $k_1\delta = 21$ and the inner-flow scaled plot shows that the region ends at approximately $k_1 z = 0.4$. Results (not presented here) suggest that a -1-law region is not possible below $z^+ = 100$. Below this value the spectra is contaminated by structures in the viscous and buffer zones leading to a hump at the high wavenumber end which becomes larger as the wall-normal distance is reduced. A limit such as this is consistent with the theory since we can argue that below this the effects of viscosity become important - making complete similarity impossible. These numbers allow us to make some predictions about future measurements. Let the highest wavenumber within the -1-law region be k_{1up} and the lowest wavenumber in the -1-law region be k_{1low}. If we want to have one decade of -1-law region then we need $k_{1up}/k_{1low} = 10$. Using the limits above we find that $z/\delta = 0.0019$. Now this value of z/δ must correspond with $z^+ > 100$ (to avoid viscous effects) so the Reynolds number required is $\delta^+ > 52500$ (corresponding approximately with $R_\theta = 137700$). This is currently beyond the capability of the wind-tunnel used in this study. It is very important to note that the length of the -1-law region depends only on z/δ with the restriction that this corresponds with a sufficiently large z^+. The point to emphasize is that high Reynolds numbers are only of use in examining this question if it is possible to make measurements very close to the wall (and of course with sufficient spatial resolution). This causes difficulty in many existing high Reynolds number facilities particularly where the largest length scale is moderate to small and the high Reynolds number is achieved by either increasing speed or changing the kinematic viscosity. In these facilities small values of z/δ correspond with very small distances from the wall.

Conclusions

Measurements in a high Reynolds number boundary layer show the existence of a short region of complete similarity in which a -1-power-law region is evident. The behaviour of the spectra is entirely consistent with the arguments used classically to derive the -1-law for large (infinite) Reynolds numbers. Examination of the length of the region and the position in which it occurs suggest that measurements must be made, not only in high Reynolds number flows, but also sufficiently close to the wall if the -1-law is to be detected. This may, in part, explain why other researchers have had difficulty in identifying this region.

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