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Periodic Forcing of Baroclinic Basin-Scale Waves in a Rotating Stratified Basin

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Abstract

A laboratory study is used to investigate the baroclinic basin-scale response to an imposed external periodic forcing. It is shown that near-resonant periodic forcing results in the generation of Kelvin and Poincaré modes of the basin which satisfy the initial condition as well as a forced response. Forcing at the resonant frequency results in the amplification of the Kelvin mode as well as higher harmonics of the forcing frequency, thus providing a mechanism by which energy may be transferred directly from a periodic basin-scale external forcing to higher frequency internal waves. Despite the resonant amplification of the Kelvin wave, there is little suggestion of nonlinear steepening of the basin-scale wavefront and the transfer of energy to higher frequency waves previously noted in the non-rotating analogue of the experimental study considered here.

Introduction

The hydrodynamics of small to medium stratified water bodies unaffected by rotation has been studied extensively (for a detailed review see [9]). Recent work has noted that a single basin-scale external forcing event (due to the wind) excites a baroclinic basin-scale wave response which exhibits nonlinear steepening of the wavefront resulting in the transfer of up to 20% of input energy to shorter wavelength waves after some time [8, 5]. These shorter wavelength waves may then propagate to the lake boundary where they break, resulting in enhanced vertical transport of pollutants in this region [13, 4]. The temporal evolution and, hence, the energy pathways of these basin-scale waves was found to be dependent on the amplitude of the initial forcing and the ambient stratification [8].

In large stratified lakes, the hydrodynamics are influenced by the Earth's rotation, so that the progressive baroclinic response to a single basin-scale external forcing event consists of basin-scale Kelvin and Poincaré waves which propagate in a cyclonic or anticyclonic fashion around the lake boundary [7, 1]. A recent laboratory study [14] demonstrated that in a circular domain, the temporal evolution of the Kelvin and Poincaré waves excited by a single forcing event was essentially linear with frictional effects at the boundaries steadily dissipating wave energy. The authors concluded that rotation acts, via Ekman dynamics, to damp the wave motion before one can observe significant nonlinear behaviour associated with the basin-scale wave.

Field observations in large stratified water bodies subject to periodic forcing by the wind have noted amplification of the gravest natural mode of the basin (a Kelvin wave in most instances), when the forcing events occur at approximately the resonant frequency [2, 11]. Resonant forcing of the wave response may result in a sufficient increase in amplitude that nonlinear wave behaviour (e.g., steepening of the wavefront, wave/wave interactions) may be observed which, in turn, would modify the energy pathways for the basin-scale waves and may provide an explanation for the high frequency waves observed in large stratified lakes [12, 4].

The objective of this paper is to use a laboratory experiment to determine the influence of periodic forcing on the energy pathways of baroclinic basin-scale waves in rotating stratified basins.

Laboratory Facility

The experiments were conducted in a 95 cm diameter cylindrical perspex tank of depth 50 cm filled with a two-layer density stratified fluid. The tank was mounted on a rotating turntable that revolved counterclockwise at a constant rate $\Omega = f/2$ (figure 1). In a typical experiment, the tank was filled with fresh water to the desired upper layer depth and allowed to spin up into solid body rotation. A saline solution was then carefully introduced beneath the lighter, fresher water until the desired lower layer depth was achieved. A semi-cylindrical perspex insert (closed at the bottom), attached via a pulley system to a small DC motor mounted on the rotating turntable frame, was initially positioned at the free surface over one half of the circular domain.

A basin-scale external forcing was modelled by oscillating the insert, using an eccentric crank and arm attached to the DC motor, so that over one forcing period the free surface over half of the tank was depressed 1.5 cm before being restored to its initial position. The observed response to such a forcing was predominantly baroclinic in nature. Periodic forcing was obtained by repeatedly driving the insert at a constant frequency of oscillation σ. The interface displacement created by forcing the free surface was sampled at 5 Hz at three positions by ultrasonic internal wave gauges [10] that were positioned along a radial transect perpendicular to the insert (figure 1).

The radius of the semi-cylindrical forcing mechanism was equal to the radius R_0 of the cylindrical tank, as shown in figure 1. The baroclinic Rossby radius of deformation is given by $R = c_0/f$ where $c_0 = (g'H_1H_2/(H_1+H_2))^{1/2}$ is the baroclinic phase speed and $g' = g(\rho_2 - \rho_1)/\rho_2$, where g is acceleration due to gravity and ρ_2 and ρ_1 is the density of the upper (H_1) and lower (H_2) layer depths respectively, thus defining the Burger number $S = R/R_0$, that provides a measure of the relative importance of stratification versus rotation [1]. The inertial frequency f and the baroclinic phase speed c_0 where varied so that S varied between 0.5 and 0.25. The total fluid depth was 20 cm while the ratio of the layer depths $H_1/H_2 = 1$. All densities were measured with a digital densimeter.

Timescales

The experimental configuration presented in figure 1 suggests three timescales that may be important in determining the baroclinic basin-scale wave response to periodic forcing.

The time taken for rotation to influence the hydrodynamics is

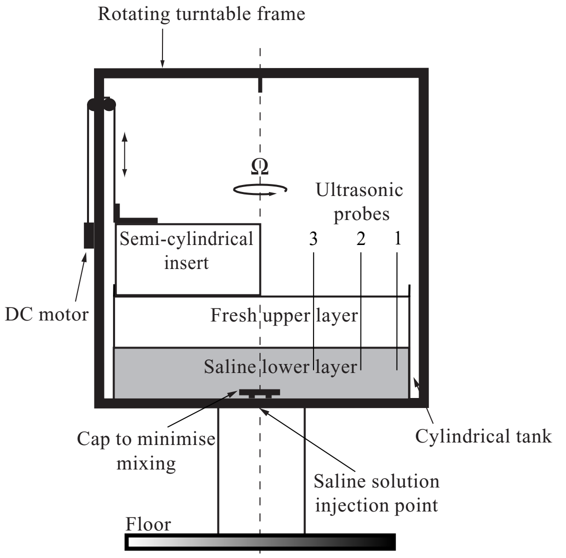

Figure 1: The experimental facility. The three ultrasonic wave gauges were positioned 3 cm, 13 cm, and 38 cm from the tank sidewall.

given by the inertial period

$$T_I = \frac{2\pi}{f}. \tag{1}$$

The closed nature of the circular basin introduces a second timescale

$$T_N = \frac{2\pi}{\omega}, \tag{2}$$

where ω is the frequency of the gravest natural mode of the basin (a Kelvin wave for the range of S considered here). This timescale characterizes the time taken for the Kelvin wave to propagate around the basin.

The periodic nature of the forcing introduces a third timescale

$$T_F = \frac{2\pi}{\sigma}, \tag{3}$$

where σ is the frequency of the forcing.

Results

Single Forcing Event

Consider first the baroclinic response following a single oscillation of the forcing mechanism (step forcing). Interface displacement and time series measured by the radial array of ultrasonic wave gauges are presented in figure 2 (a) for $S = 0.5$. The transient response to the forcing event is similar to the laboratory experiments of [14], consisting of multiple frequencies which decay in time with no wave motion evident after 30 T_I. Comparison of time series between positions 1 and 3 clearly shows that the interface displacement amplitude decays offshore. Power spectra of the interface displacement time series in figure 2 (a) is shown in figure 2 (b). Two waves of significance can be identified for this run: a sub-inertial (Kelvin) and a super-inertial (Poincaré) wave with non-dimensional frequencies (ω/f) of 0.55 and 1.51 respectively.

The frequencies of the natural modes for a circular basin containing a two-layer stratification can be predicted using the

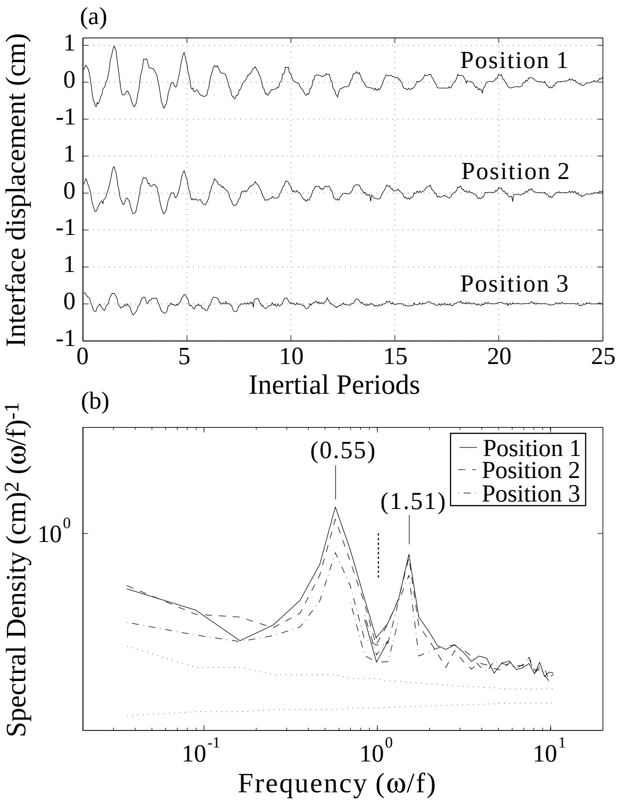

Figure 2: (a) Time series of the interface displacement for $S = 0.5$ following a step forcing. (b) Power spectra of the interface displacements shown in (a). The wave frequency ω is scaled by f with the dashed vertical line identifying the inertial frequency f. Spectra have been smoothed in the frequency domain to improve confidence, with the 95% confidence level is given as the difference between the two dotted lines at a prescribed frequency [3].

dispersion relations derived by [7]. For $S = 0.5$, the gravest mode of the basin is an azimuthal mode one, radial mode one Kelvin wave predicted to have a non-dimensional frequency of 0.65 while the azimuthal mode one, radial mode one Poincaré wave has a non-dimensional frequency of 1.51. The good agreement between observed and predicted frequencies suggests (see also [14]) that such a frequency comparison can be used to identify the structure of the basin-scale modes observed following a single forcing event

Periodic Forcing

The baroclinic response subject to continuous forcing is illustrated in figure 3. Comparing the response to a step forcing presented in panel (a) with that observed for periodic forcing shown in panel (b) it is clear that there is an amplification of the interface displacement with the wave period being determined by the forcing period T_F. Once the forcing has ceased, however, the wave period is determined by the period of gravest (Kelvin) mode of the basin T_N and the rate of internal wave decay is comparable to what is observed following a single forcing event. Moreover, the baroclinic response appears to behave in an analogous fashion to a damped mass-spring system. In particular, when the forcing period T_F is similar to the period of gravest (Kelvin) mode of the basin T_N, there is evidence of wave beating during continuous forcing (see figure 3 (b)). When the

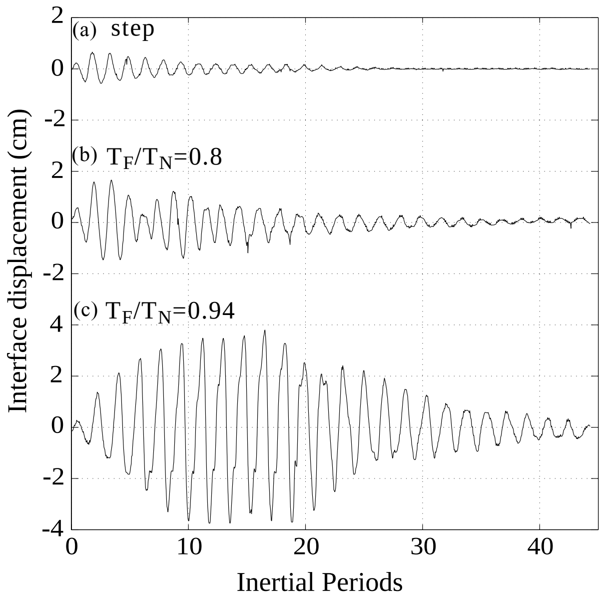

Figure 3: Time series of interface displacement recorded at position 1 for $S = 0.5$ subject to (a) a step forcing (b) 10 forcing events with $T_F/T_N = 0.8$ (c) 10 forcing events with $T_F/T_N = 0.94$.

non-dimensional forcing period $T_F/T_N \approx 1$, resonant forcing is observed, with a noticeable amplification of the interface displacement which reaches a steady state after approximately 5 forcing events and is sustained for the duration of the periodic forcing event (see figure 3 (c)).

Figure 4 demonstrates the amplification of the interface displacement as $T_F/T_N \to 1$. Note that maximum amplification is achieved when the dimensionless forcing period is slightly less than 1 ($T_F/T_N = 0.94$), due to the influence of frictional effects, which is consistent with analytically derived results for the classical damped mass-spring system forced at resonance [6]. As noted earlier, frictional dissipation by complicated Ekman dynamics plays an important role in determining the hydrodynamics in the current experimental configuration [14].

Now consider the power spectra of interface displacement time

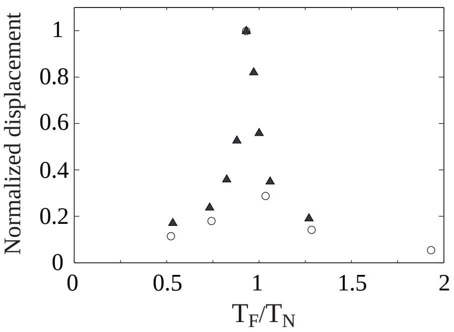

Figure 4: Normalized interface displacement recorded at position 1 as a function of the dimensionless forcing period T_F/T_N for $S = 0.5$ (filled triangles) and $S = 0.25$ (circles). Steady state interface displacements were measured after 20 forcing events and are normalized by the interface displacement at resonance $T_F/T_N = 0.94$.

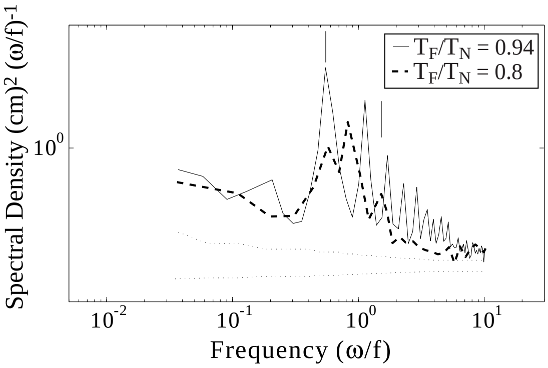

Figure 5: Power spectra of the interface displacement recorded at position 1 for $S = 0.5$ subject to 20 forcing events with $T_F/T_N = 0.8$ and $T_F/T_N = 0.94$. The solid vertical lines indicate the frequency of the natural modes of the basin (see figure 2 (b)) which satisfy the imposed initial condition. Spectra were smoothed in a similar manner to figure 2

series following near-resonant ($T_F/T_N = 0.8$) and resonant ($T_F/T_N = 0.94$) periodic forcings presented in figure 5. As noted earlier in figure 2, two modes of the basin are observed following a step forcing: a Kelvin and Poincaré wave. When $T_F/T_N = 0.8$, these natural modes of the basin are observed but there is also a third peak which dominates the power spectra corresponding to the forcing frequency at $\omega/f = 0.7$.

When forcing at the resonant frequency, the dominant spectral peak is now associated with the Kelvin wave (the frequency of which now coincides with the forcing frequency) but at least 4 other spectral peaks can be clearly identified with non-dimensional frequencies of 1.1, 1.66, 2.24, and 2.80, respectively. The non-dimensional forcing frequency is 0.55 (the Kelvin wave frequency), suggesting that the higher frequency waves are harmonics of the resonant forcing frequency.

The interface displacement time series observed when forcing at the resonant frequency can be bandpass filtered to isolate the contribution of the Kelvin wave and the higher frequency harmonics. The filtered signals for the Kelvin wave and the first harmonic of the forcing frequency are presented in figure 6. It is evident that forcing at the resonant frequency not only results in amplification of the gravest mode of the basin but also essentially simultaneously the first harmonic until a steady state interface displacement is achieved which is then sustained until the periodic forcing ceases.

Nonlinear steepening of the basin-scale wavefront and the subsequent transfer of energy to higher frequency waves after some time, previously noted in non-rotating stratified basins, does not readily explain these observations due to the fact that energy resides in the higher frequency waves almost immediately after initiation of the resonant forcing (see figure 6 (b)). Instead, the results presented here suggest that energy may be transferred directly from the basin-scale external forcing to higher frequency internal waves which are harmonics of the resonant forcing frequency.

Conclusions

We have investigated the baroclinic basin-scale wave response to periodic forcing in a rotating stratified basin. It was demonstrated that the response was analogous to a damped string-mass system with continuous forcing at near-resonant forcing resulting in the generation of Kelvin and Poincaré modes of the basin

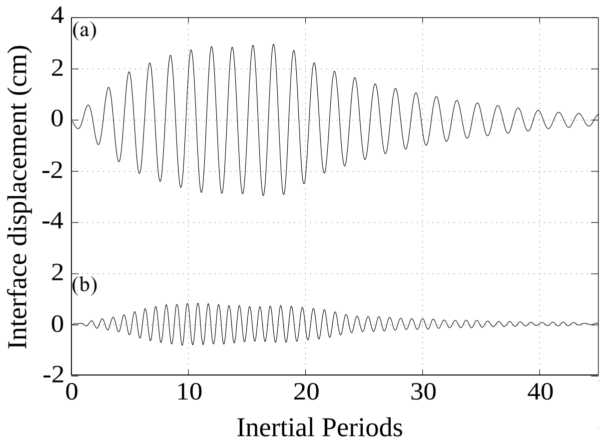

Figure 6: Bandpass filtered time series of interface displacement recorded at position 1 for $S = 0.5$ subject to 10 forcing events with $T_F/T_N = 0.94$ for (a) the Kelvin wave, (b) first harmonic of the forcing frequency.

which satisfy the initial condition as well as a forced response. Forcing at the resonant frequency resulted in the amplification of the Kelvin mode as well as higher harmonics of the forcing frequency thus providing a mechanism by which energy may be transferred directly from a periodic basin-scale external forcing to higher frequency internal waves. The exact nature of this transfer mechanism remains to be determined and will be the focus of subsequent studies. Despite the resonant amplification of the Kelvin wave, there was little suggestion that this mechanism was associated with nonlinear steepening of the basin-scale wavefront and the transfer of energy to higher frequency waves.

Acknowledgements

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