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The Entrance Length for Fully Developed Turbulent Channel Flow

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Abstract

An experimental investigation into the entrance length for fully developed turbulent flow in a smooth channel was undertaken. The Reynolds number based on channel height were approximately 40×10^3, 105×10^3, and 185×10^3. Mean velocity profiles were taken with a Pitot-static tube between distances of 70 and 205 heights from the inlet. A vertical shift—the direct result of a change in skin friction—is found to be the most salient indication of under-development in the velocity profile. It is suggested that commonly used criteria for fully developed flow may be inadequate to detect these first instances of under-development.

Nomenclature

a	channel half-height
A	universal constant
C_f	skin friction coefficient
d_p	Pitot-tube diameter
h	channel height (internal)
K_τ	Karman number: $K_\tau = U_\tau h/\nu$
p	static pressure
Re	Reynolds number: $Re = \overline{U}h/\nu$
U	local mean streamwise velocity
$\overline{U}$	mean/bulk streamwise velocity
U^+	non-dimensional U: $U^+ = U/U_\tau$
U_{CL}	centreline velocity
$U_{\max}$	maximum streamwise velocity
U_τ	friction velocity: $U_\tau = \sqrt{\tau_w/\rho}$
x	distance in streamwise direction from channel inlet
y	distance normal to the channel wall; wall distance
y^+	non-dimensional wall distance: $y^+ = yU_\tau/\nu$
δ	boundary layer thickness
η	non-dimensional wall distance: $\eta = y/a'$
κ	universal Karman constant
ν	kinematic viscosity
ρ	air density
τ_w	wall shear stress: $\tau_w = \frac{1}{2}\rho\overline{U}^2 C_f$

Introduction

Duct flow is commonly encountered in engineering. The precise nature of the evolution of turbulent flow through a duct, however, requires further investigation despite over a century of research. Any further understanding of the theoretical mechanisms that underlie such a phenomenon must necessarily rely on observation. But, for observation to be accurate, experiment must be exact.

The most basic requirements for a rigorous study of fully developed two-dimensional channel flow is clearly a knowledge of the channel dimensions necessary for its establishment. The minimum aspect ratio of 7:1 postulated by Dean [3] has provided experimentalists with a reliable standard for the achievement of nominally two-dimensional flow. By contrast, there is a distinct scarcity of thorough investigations on the minimum entrance length for fully developed flow—probably due to the many challenges such a study encounters. Efforts to compile information from existing literature have revealed data that is scattered and unreliable. Present-day experimentalists are still without a definitive guide to the necessary length for an intended experimental facility.

One such effort to compile channel flow data was made by Dean [3], who noted that references to the entrance length were generally vague, if not entirely omitted—which is surely symptomatic of the inadequacy of the current state of knowledge. The lengths (from inlet to measuring station) that he did find ranged widely from 23 to 300 heights. Notable among Dean's sources is the seminal work of Laufer on turbulent channel flow, which was performed at a length of some 55 heights. His later work on turbulent pipe flow [4] claimed full flow development at 30 diameters based on "the measured mean velocity distribution" (p.421). Similarly, in his paper on turbulent pipe flow, Nikuradse [7] concluded from a comparison of mean velocity profiles at successive streamwise lengths that the flow was fully developed by 40 diameters.

No absolute minimum length has been established, but the lengths of Laufer and Nikuradse were substantially exceeded by a large number of those surveyed by Dean. Furthermore, none of the more recent investigations that have employed more stringent criteria have been found to claim entrance lengths much lower than 100, casting doubts on the state of the flow in these classical works. Patel [8], for example, in a dedicated study of entrance length, found a minimum length of 100 pipe diameters for sufficiently high Reynolds numbers. Zanoun, Durst, and Nagib [10] claim a sufficient channel length at 115 heights based on the oft-cited work of Comte-Bellot. Zagarola [9] applied numerous checks to find a pipe entrance length of 160 diameters was adequate for all but the lowest Reynolds number.

This investigation aims to help fill what appears to be a critical gap in the current knowledge of turbulent duct flow.

The Test Facility and Experimental Equipment

The channel flow facility (shown in Figure 1) is situated in the Walter Bassett Laboratory at the University of Melbourne. With a width of 1170 mm and a height of 100 mm giving an aspect ratio of 11.7:1, the existence of a region of nominally two-dimensional flow in the channel is ensured. Spanwise measurements of mean flow quantities in the channel by Monty, Jones and Chong [6] have since confirmed this. Furthermore, at 22.5 m (or 225h), the length of the channel working section was considered to be ample for the attainment of fully developed flow.

The air flow is driven by a centrifugal fan capable of volume flow rates of up to 3.51 m^3/s. A diffuser and a settling chamber, which house a honeycomb screen and a series of 12 fine-mesh screens, serve to straighten the incoming air and minimise turbulence at the channel entrance. The two-dimensional, 9:1 area ratio contraction—achieved by splicing together a cubic and quadratic curve—is designed to avoid flow separation and deliver uniform flow to the channel inlet (at $x/h = 0$). The effectiveness of the facility to this point in producing the desired uniform flow was verified by mean velocity profiles at a number

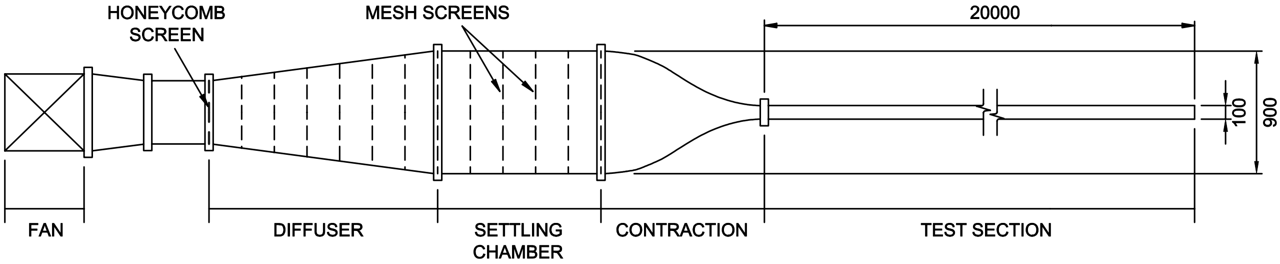

Figure 1: The channel flow facility (side view).

of spanwise locations. At the inlet the entire perimeter is lined with a 100-mm wide strip of 80-grit sand paper to promote immediate transition to turbulence, thereby shortening the length required for the flow to reach a fully developed state.

The channel itself is constructed from $1220 \times 2440 \times 18$ mm Medium Density Fibreboard (MDF) that has been repeatedly varnished and sanded and, finally, waxed on one side to achieve the required finish for a smooth wall investigation. The floor boards are supported against deflection under load by three 6-mm thick extruded aluminium alloy C-section joists. Two 3-mm thick aluminium alloy C-section beams were bolted between the channel floor and ceiling boards to serve as the channel side walls. Wooden stiffeners glued onto the upper side of the channel at regular intervals constrain any deflection of the channel ceiling to within 0.5%.

Six measuring station were successively installed along the centreline of the completed channel at a streamwise distances of $x/h = 205$, 176, 148, 128, 94, and 70. Four stainless steel pressure taps were inserted into the upper surface of the channel along the centreline at intervals of approximately $25h$, starting at $x/h = 120$. The finished taps were inspected for burrs and other defects, and were tested against a removable static pressure probe before use.

All mean velocity measurements for this investigation have thus far been performed using a stainless steel Pitot-static tube with an outer diameter of 1 mm. The Pitot-static tube was connected to a MKS Baratron pressure transducer. The output from the transducer was sampled by a computer equipped with a 16-bit Microstar 4000a Data Acquisition Processor (DAP) board.

Experimental Procedure and Data Analysis Methods

Measurements

Mean velocity profiles were taken at each of the six stations for Reynolds numbers of approximately 40×10^3, 105×10^3, and 185×10^3. Air temperature and pressure were measured before and after the collection of each mean velocity data set. Their respective values were adjusted when necessary. The fully developed streamwise pressure drop, dp/dx, was obtained by applying a linear curve fit to readings from the four wall-mounted pressure taps. Scatter within 1% was considered acceptable. This was also done before and after each data set.

Wall Distance

The initial wall distance was found by first traversing the Pitot tube down until it pressed against the channel wall. It was then traversed back upwards an incremental distance of 0.05 mm, where a reading was taken and compared to the previous reading. This process was repeated until a noticeable difference between two consecutive readings was seen. This reading was then taken to correspond to a wall distance of $y = 0.05$ mm. This method relies on the specification of a rather arbitrary threshold value and was not found to be reliably precise (with an error of ± 0.05 mm). The effect of this inaccuracy in the measurement of the wall distance, however, diminishes with each successive y value, such that the impact of the error on the velocity profile away from the wall is marginal.

Mean Flow Parameters and Non-dimensional Quantities

The local mean velocities are calculated from the Pitot-static tube data, and the MacMillan correction for shear ($\Delta y = 0.15 d_p$) [5] is applied to the wall distances. The bulk velocity, $\overline{U}$, is then evaluated from a numerical integration of the velocity profile, which thereby enables the computation of the Reynolds number, Re, based on channel height. The wall shear stress, τ_w, is calculated for *fully developed*, two-dimensional channel flow from a momentum balance equation:

$$\tau_w = -\frac{h}{2}\left[\frac{dp}{dx}\right], \tag{1}$$

which has been validated by Monty *et al.* for this channel in [6]. From this, values for the friction velocity, U_τ, and the Karman number, K_τ, for fully developed flow are obtained.

Now, velocity defect for internal flow is conventionally given by $(U_{\mathrm{CL}} - U)/U_\tau$, where U_{CL} for fully developed channel flow would correspond to a wall distance equal to largest length scale, $a = h/2$, and be equivalent to $U_{\max}$. The case of developing channel flow, however, is essentially a study of boundary-layer flow: the channel centreline is no longer necessarily an axis of symmetry, and the equivalence of U_{CL} to $U_{\max}$ is no longer guaranteed. As consistency demands that the velocity defect vanishes at $\eta = 1$, $U_{\max}$ is chosen as a more appropriate and general reference velocity, and η is accordingly redefined: $\eta = y/\delta$, where $y = \delta$ when $U = U_{\max}$ such that δ is comparable to the local mean thickness of the developing boundary layer.

Local Wall Shear Stress: The Clauser-Plot Method

The wall shear stress for the *developing* flow is determined by the Clauser-plot method [2], which is based on the assumption of the existence of a universal logarithmic region for constant pressure turbulent boundary layer flow. Here, the section of the velocity profile described by the Prandtl's law of the wall (the inner flow region) overlaps that described by von Karman's velocity-defect law (the outer flow, or core, region). This overlap region can be represented in terms of inner-scaled and outer-scaled variables, respectively, by

$$U^+ = \frac{1}{\kappa}\ln y^+ + A \tag{2}$$

and

$$\frac{U_{\max} - U}{U_\tau} = -\frac{1}{\kappa}\ln\eta + B, \tag{3}$$

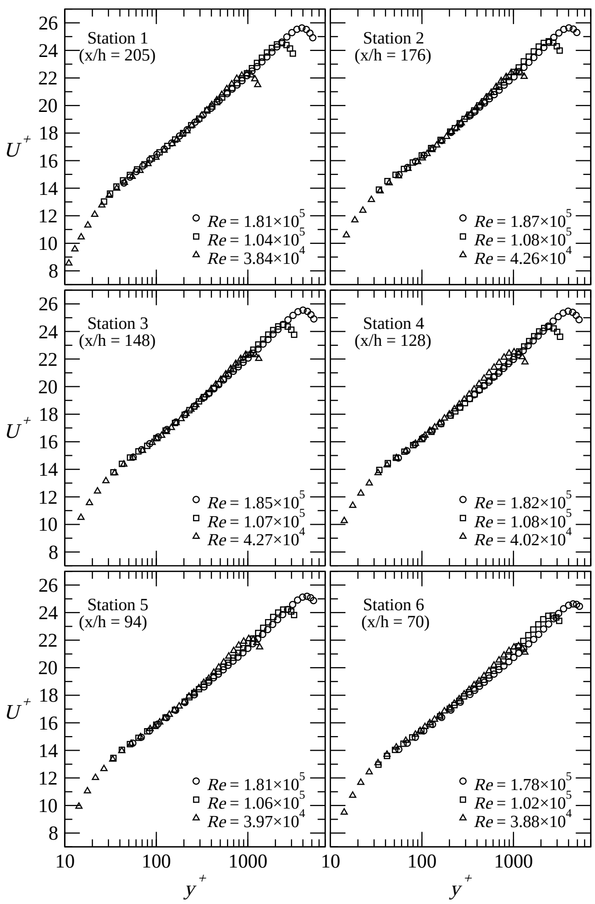

Figure 2: Mean velocity profiles, where y^+ and U^+ are scaled with the fully developed U_τ. Profiles are shown for $Re \approx 40 \times 10^3$, 105×10^3, and 185×10^3, at each of the six stations.

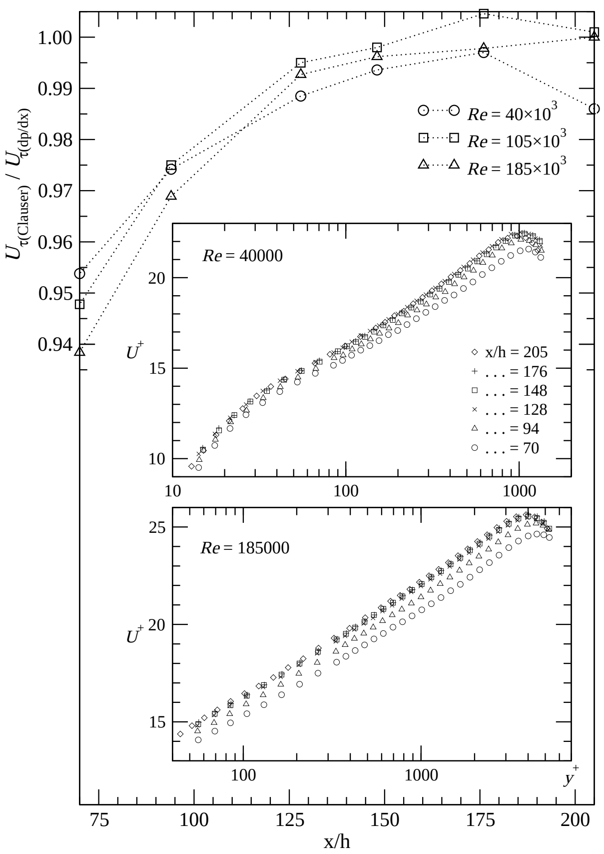

Figure 3: Ratio of U_τ for developing flow to that of the fully developed flow, for $Re \approx 40 \times 10^3$, 105×10^3, and 185×10^3. Inset plots show the mean velocity profiles, where y^+ and U^+ are scaled with the fully developed U_τ. Profiles are shown for $Re \approx 40 \times 10^3$ (top) and 185×10^3 (bottom) at all six stations.

where A and κ are universal constants, and B is dependent on large-scale flow geometry. The Clauser values of U_τ were used as an alternative scaling for the mean velocity data, some of which will be presented in the following section.

Results and Discussion

Figure 2 shows the mean velocity profiles for the three Reynolds numbers from each of the six stations. This data is found to be consistent with previous unpublished results in this facility by Monty (Monty, J.P., 2004, pers. comm.). The inner flow variables y^+ and U^+ in this figure remain scaled with U_τ found by Eq. (1). Because it is unlikely that the flow is fully developed at the more upstream stations, this scaling means that the collapse of all of the inner flow data is not ensured. Nevertheless, these plots show that the data is in good agreement with the general form of the inner-scaled logarithmic law; the existence of the universal overlap region is well supported. That inner flow similarity is still apparent for data from any single given station suggests that dependence of entrance length on Reynolds number is weak.

Also notable in Figure 2 is a small, but discernible increase in the size of the wake as the inlet is approached—particularly at the lowest Reynolds number. What is perhaps not clear, however, is the vertical shift of the profiles that results from the use of the fully developed value of U_τ. The subplots in Figure 3 show the superposition of velocity profiles from all stations for the respective values of $Re \approx 40 \times 10^3$ and 185×10^3. The use of appropriate velocity scales would see all the data collapse onto a single profile. Instead, only the profiles from stations downstream of a distance just short of $x/h \approx 128$ (i.e., stations 1 to 4) appear to do so. Profiles taken below this channel length are increasingly shifted downwards as the inlet is approached.

It should be remarked that this is a surprising finding as it was expected that, as the inlet was approached, under-development would first be evidenced in a change in the structure of the wake. As such, the first instance of vertical shift prompted a systematic inspection of the equipment and repeated measurements in search of a source of error. Subsequent measurements have firmly established an undeniable pattern that is certainly not a product of error. It is thus suggested that the inappropriateness of the velocity scale implied by the vertical shift in the profiles is indicative of the changing wall shear stress in the entrance region of the channel. The actual values of the friction velocity that enable the collapse of the profiles (found via the Clauser-plot method, taking $\kappa = 0.395$ and $A = 4.65$ from the aggregated station 1 data) were plotted as a ratio to the fully developed friction velocity in Figure 3. This ratio is proportional to the observed vertical shift.

Though the scale on this plot is greatly magnified, a slight, but definite increasing trend in the shear stress is evident as the flow moves along the channel towards full development. It is encouraging that the trend seen here towards the end of the entrance region would seem to be in accord with that observed by Byrne, Hatton, and Marriott [1]. Their measurements, spanning from channel inlet to some 36 heights for Reynolds numbers ranging from 100,600 to 221,000, suggest an initial decrease in shear

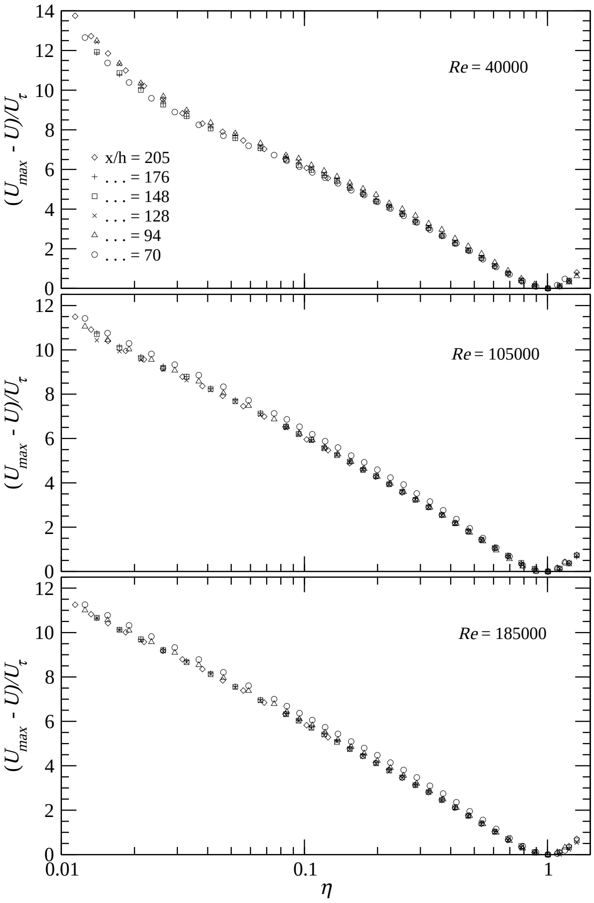

Figure 4: Velocity-defect profiles, where η and U_{def} are scaled with local U_τ obtained by the Clauser-plot method. Profiles are shown for $Re \approx 40 \times 10^3$ (top), 105×10^3 (middle), and 185×10^3 (bottom) at all stations.

stress within the first 20 heights, wherein the fully developed value is overshot, before a final increase to the constant value. Significantly, no clear dependence of shear stress development on Reynolds number is observed from the data in Figure 3.

Upon closer inspection of Figure 3, it may appear that constancy of the shear stress is not convincingly achieved. Consider, however, that experimental error in the calculated value of U_τ from Eq. (1) is of the order of $\pm 0.5\%$. In addition, the error associated with the use of the Clauser chart may be greater, especially at the lowest Reynolds number (as can be seen from the plot) where as few as three points may occur in the logarithmic region. Hence, while it is certain that the shear stress, and therefore the flow, is still developing up to $x/h \approx 130$, nothing conclusive can be inferred from this plot beyond that length.

The under-development of the flow at stations 5 and 6 suggested by the above inner-scaled profiles is substantiated in Figure 4. Their properly scaled velocity-defect profiles—at least those of the two higher Reynolds numbers—are found to increasingly diverge from those of the other stations away from $\eta = 1$, which is particularly reflective of the larger wakes seen earlier. (It has yet to be determined whether the failure of the station 6 data at $Re \approx 40 \times 10^3$ to follow this trend is a genuine phenomenon, or simply a matter of experimental error; though the latter is suspected.)

Conclusions and Further Research

This preliminary study shows that under-development in relation to the velocity profile is first manifested as a vertical shift, which is likely to be indicative of a change in the skin friction. Tests for fully developed flow that seek changes in the wake structure of the velocity profile may not detect the earliest instances of under-development. Moreover, testing for fully developed flow commonly involves two measurements over a distance that is substantially shorter than that separating the measuring stations used in this investigation. It was seen that even at these intervals observable changes in the mean velocity profiles are slight; shorter intervals may therefore not be adequate to resolve the vertical shift seen here with any certainty.

It is estimated from the present data that a minimum length of 130h is required for flow to become sensibly constant with streamwise direction, although a more conservative entrance length of 150h could be recommended.

Mean velocity profiles alone, however, would not constitute a thorough study of the development of turbulent channel flow. Measurements of turbulence quantities by hot-wire anemometry are underway.

Acknowledgments

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