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The Effect of Turbulence on Cloud Droplet Collision Rates

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Abstract

Direct numerical simulations of an evolving flow field have been performed to explore how turbulence affects the motion and the collisions of cloud droplets. Large numbers of droplets are tracked through the flow field and their positions, velocities and collision rates have been found to depend on the eddy dissipation rate of turbulent kinetic energy. The radial distribution function, which is a measure of the preferential concentration of droplets, increases with eddy dissipation rate. The clustering is most pronounced for 15 and 20 μm droplets, which is the largest radius ratio considered, increasing to more than twice the value for a homogeneous Poisson distribution. This increase from the value for sedimenting droplets in a quiescent flow, together with an increase in the mean radial relative velocity of colliding droplets, leads to increases in the geometric collision kernel of up to almost 4 times the corresponding gravitational kernel. The greatest increase in the collision efficiency is observed for the smallest radius ratio. Interacting 5+20 μm droplets collide almost 3 times more frequently when turbulent accelerations are included in their equation of motion as compared to the purely gravitational case. The lowest dissipation rates had the greatest effect on the collision kernel that includes the hydrodynamic forcing between interacting droplets for the smallest radius ratio considered. The most energetic flow field produced the largest increase in this kernel for the droplets of 15 and 20 μm in radius and this was the most significant increase observed of more than 9 times the gravitational kernel. These increases are expected to play an important role in the broadening of the drop size distribution and the initiation of rain.

Introduction

The time of transition from cloud droplet growth by condensation to that of effective collision and coalescence is an outstanding issue in cloud physics. Turbulence has long been postulated to reduce the time it takes to produce rain drops by accelerating the growth of droplets in the size range of 5 to 30 μm in radius. Due to the complexity of the problem, previous studies of turbulence-droplet interactions made use of statistical techniques and numerous assumptions about the flow field and the motion of the droplets and hence there is a lot of discrepancy between the results. With the increase in computational power we are now in a position to be able to use direct numerical simulations (DNS) of the flow field to investigate the effect of turbulence on the collisions of droplets. DNS have the advantage of explicitly resolving the dissipation range dynamics, which has been shown to have the most influence on the local particle accumulation and settling rate of small particles whose diameter is less than that of the small eddies of the flow [9].

As demonstrated by Sundaram and Collins [6] the collision kernel can be expressed by two statistical quantities of the particle phase; the radial distribution function *g(R)* and the particles' relative velocity $|\boldsymbol{w}|$. When an inertial particle interacts with a turbulent structure, because of centrifugal forces the particles will tend to cluster or become preferentially concentrated in regions of the fluid with low vorticity and increased pressure in the boundaries between eddies. Since an increased local particle density means that there is an increased probability of finding two droplets closely separated in the flow, consequently there is an increase in the collision rate or kernel. Wang *et al.* [10] extended this work and demonstrated that the spherical geometry is the correct form to use in the problem of collision in a turbulent flow field and thus the collision kernel, Γ, for droplets of radius r_1 and r_2 is given by

$$\Gamma = 2\pi R^2 \left\langle |w_r| \right\rangle g(R) \qquad (1)$$

where R is the sum of the droplet radii and the radial relative velocity w_r is defined as $\boldsymbol{w}\cdot\mathbf{R}/|\mathbf{R}|$, where $\mathbf{R}$ is the separation vector for the droplets' positions. For droplets falling only under the force of gravity, the collision kernel is expressed by (1) with $g(R)$ equal to 1 and $<|w_r|>$ equal to half of the difference in the terminal velocities of the droplets. The collision kernel can also be defined in terms of the direct counting of collisions as

$$\Gamma = n_c \Omega / dt N_1 N_2 \qquad (2)$$

where n_c is the average number of collisions per time step, Ω is the volume, dt is the time step and N is the number of droplets from the respective size group.

Numerical Methods

The turbulent flow field is generated by solving the Navier-Stokes equations using a pseudo-spectral model. For a detailed description of the model see Vaillancourt *et al.* [8]. The initial kinetic energy is specified and throughout the simulation energy is forced into the large scales of the flow to produce a statistically stationary field. Four numerical simulations are performed with the number of grid points N^3 equal to 80^3, 120^3, 180^3 and 240^3. The computational domain is fixed for all experiments at 10x10x10 cm^3, thus we increase in spatial resolution from a grid length of 0.125 cm to 0.04166 cm. Periodic boundary conditions are imposed in all three directions. The flow field that is generated by the model is homogeneous and isotropic and this structure is expected to be representative of the small-scale flow of adiabatic cloud cores. It is this region of the cloud where one would expect to find the large droplets needed to initiate effective collision-coalescence growth, as the main body of the cloud typically has lower water contents. The average eddy dissipation rates, ε, for the simulations are 95, 280, 656 and 1535 $cm^2\ s^{-3}$, which span the range of observed values in cumulus clouds. Even though the Reynolds numbers of the flows (R_λ increases from 33 to 55) are much smaller than those of atmospheric turbulence, the Kolmogorov scales are close to atmospheric values and it is these scales that have the most influence on droplet motion.

25 000 droplets of each of the two sizes being considered are tracked in the simulations. Due to the small volume fraction occupied by droplets we neglect any modification that the droplets may have on the turbulence and only consider binary collisions. The equation of motion for the droplets is

$$\frac{d\mathbf{V}(t)}{dt} = \frac{1}{\tau_p}\left(\mathbf{U}[\mathbf{X}(t),t] - \mathbf{V}(t)\right) + \mathbf{g} \tag{3}$$

where **V**(t) is the droplet velocity at time *t*, **U** is the flow field, **X** is the droplet position, **g** is gravity and τ_p is the droplet inertial response time based on the Stokes drag force, which is a function of the droplet mass, size and the dynamic viscosity of air. The droplets are randomly distributed in the domain once the turbulent flow field has reached a statistically stationary state. To ensure there is no influence of the initial conditions on the results, the droplet positions and velocities are allowed to evolve in time before the calculation of the collision statistics begins. For a description of the collision detection scheme see Franklin *et al.* [2].

Geometric Collision Kernels

The collision statistics have been calculated for collisions between a collector droplet of 20 μm in radius and droplets of sizes 5, 10 and 15 μm. These sizes have been chosen as it is collisions between droplets in the range of 5 to 30 μm that are needed to allow some large droplets to grow to a size that will initiate effective coalescence. If turbulence is to significantly reduce the time it takes to produce rain drops, the collision kernel for small droplets must increase from the gravitational collision kernel. Figure 1 shows the sensitivity of the collision kernels to the turbulent forcing. The turbulent kernels calculated from (2) for the 3 droplet size combinations all show a fairly linear response to increasing dissipation rates. For the lowest dissipation rate the turbulent kernels are only marginally larger than the corresponding gravitational kernel. However, as the dissipation rate increases the turbulent kernels increase, with the largest radius ratio (15+20 μm) giving the greatest increase of almost 4 times the gravitational case. These increases are significant and are expected to influence the evolution of the drop size distribution. The physical mechanisms responsible for these increases are discussed in the following section.

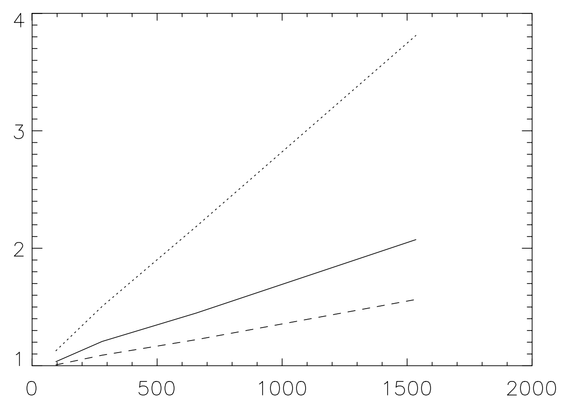

Figure 1. The turbulent collision kernel normalised by the corresponding gravitational kernel as a function of average eddy dissipation rate (cm^2 s^{-3}). The dotted line is the normalised collision kernel for the 15 and 20 μm droplets, the solid line is for the 10 and 20 μm droplets and the dashed line is for the 5 and 20 μm droplets.

Radial Relative Velocity and Preferential Concentration

As shown in (1) the collision kernel is partly a function of the relative velocity between droplets. Just as the differential inertia effect due to gravitational acceleration defines the relative velocity in a quiescent flow, this effect also contributes to the relative velocities amongst droplets in a turbulent flow. The turbulent flow field can change the droplets' velocities by two physical processes. The first is the inertial bias which causes droplets to accumulate in the peripheries of local vortical structures. The second process is caused by the droplets tending to move on the downflow sides of these vortices due to particle inertia, the local velocity field and the way the droplets generally approach these structures from above [9]. The quantity that is input into the collision kernel calculation is the mean radial relative velocity of the droplet pairs that are separated by a slightly larger distance than that required for a collision. The reason being that the non-uniform flow field results in a local non-uniform relative velocity distribution [11]. The other kinematic quantity that can change the collision kernel is the preferential concentration of the droplets. In a bidisperse system the clustering is determined by the correlation between particle concentrations of the two size groups. Zhou *et al.* [12] demonstrated that particles of different sizes will tend to cluster in different regions of the flow field due to the different inertial responses to the flow accelerations.

Table 1 and figure 2 show that for small radius ratios, that is the 5+20 μm case, the increase in the turbulent collision kernel that is illustrated in figure 1 results mostly from increases in the radial relative velocities between droplets. The spatial distribution of these droplets in the two lowest dissipation rate flows show no spatial correlations, as *g(R)* is close to 1 which is the value for a homogeneous Poisson distribution. For the two highest dissipation rates, the droplets become preferentially concentrated with a maximum value of *g(R)*=1.1296, which demonstrates that in this case the chances of seeing two droplets, one from each size category, closely separated in the flow is approximately 13% greater than the purely gravitational case.

	ε	**95**	**280**	**656**	**1535**
5+20	$\langle\lvert w_r\rvert\rangle_n$	1.0120	1.0684	1.1624	1.3500
	g(R)	0.9905	1.0023	1.0517	1.1296
10+20	$\langle\lvert w_r\rvert\rangle_n$	1.0119	1.0630	1.2214	1.5099
	g(R)	1.0274	1.1949	1.2311	1.3672
15+20	$\langle\lvert w_r\rvert\rangle_n$	1.0181	1.1169	1.3283	1.7801
	g(R)	1.1158	1.3966	1.6912	2.1679

Table 1. Preferential concentration *g(R)* and the normalised mean radial relative velocity $\langle\lvert w_r\rvert\rangle_n$ between droplets that are closely separated in the domain, for the three size combinations of droplets in the four different flow fields with increasing dissipation rates ε (cm^2 s^{-3}). The velocities have been normalised by the corresponding gravitational values.

For collisions between the droplets of 10 and 20 μm in radius, the increase in the collision kernel is comparable between the two kinematic quantities across the four simulations. For the largest radius ratio, the 15 and 20 μm case, the large increases shown in figure 1 can be attributed to increases predominately in the clustering of the droplets. However, w_r also shows the largest increase from the corresponding gravitational value for this size combination of droplets. The reason why turbulence has the greatest impact for this case is primarily because of the droplet Stokes number. The Stokes number is the ratio of the particle inertial response time τ_p, to the Kolmogorov time scale of the flow. The effect of turbulence has been demonstrated to have the most effect on *g(R)* for both non-sedimenting and sedimenting monodisperse droplets when this number is of order 1 [6,9]. The Stokes numbers of cloud droplets are small [7] and the largest Stokes number in our simulations is 0.48, which is the value for the 20 μm droplet in the most energetic flow. The trend for the effect of turbulence on the droplet clustering to increase with increasing Stokes number is demonstrated by the results in table 1 and shows support for the hypothesis of Kolmogorov scaling for the radial distribution function. It is important to note however, that the Stokes number does not take into consideration the effect of the terminal velocity of the droplets. The gravitational forcing means that the time of interaction between an eddy and a particle will be reduced and thus the Stokes number does not describe all aspects of the interaction between droplets and the turbulent flow field. This is clear from the lack of Stokes number scaling for the normalised radial relative velocities. Results for other size combinations of droplets are necessary before an appropriate nondimensional parameter to describe the interaction can be determined.

Collision Efficiencies

When two droplets approach one another, the fluid they push aside as they move through a medium interacts with the other droplet and tends to prevent the droplets from colliding. As the Reynolds numbers of the droplets in this study are quite small, with the largest value being 0.1, we can use the Stokes flow as a good approximation to represent the hydrodynamic flow field and as a consequence neglect any wake effects. To determine the relative motion between two interacting droplets we use the superposition method whereby the flow field of one drop is superimposed onto the flow field of the other drop. In this method each droplet is assumed to be influenced by the fluid motion around the other droplet in isolation. Details on the implementation of the Stokes disturbance flow solution to a three-dimensional framework are given in Franklin *et al.* [2].

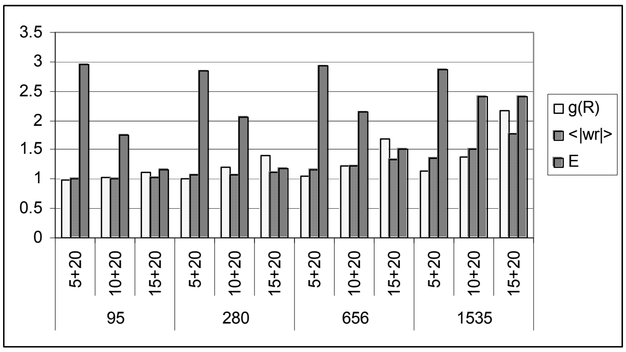

Figure 2. The contribution of the three effects (the radial distribution function *g(R)*, the radial relative velocity $\langle|w_r|\rangle$ and the collision efficiency *E*), to the increase in the turbulent collision kernel. The quantities have been normalised by the corresponding gravitational values. The results are presented as a function of average eddy dissipation rate (cm^2 s^{-3}) for the three size combinations.

To test the algorithm developed to calculate the hydrodynamic forces between interacting droplets, we first applied it to the case of droplets only falling under the force of gravity. The collision efficiency is defined to be

$$E = \frac{no.of\ collisions\ with\ hydrodynamics}{no.of\ collisions\ with\ no\ hydrodynamics} \qquad (4)$$

The turbulent collision efficiencies have been calculated by taking the background flow field to be fixed over the time of interaction. This methodology was adopted by Pinsky *et al.* [4], who showed that the turbulent flow evolves over much longer time and length scales compared to those of the hydrodynamic interaction. The initial separation distance is prescribed to be 20 times the radius of the largest droplet, 0.04 cm, as this is the typical value used in previous theoretical studies. The results were calculated by taking the positions and velocities of all of the collision pairs in the previous geometric collision kernel experiments with no hydrodynamic forces. The colliding droplets were interpolated backwards in time until the drops were separated by the prescribed distance and then the drops were allowed to move towards one another with their velocities being modified by the forces of gravity, the constant background velocity of the turbulent flow field and the hydrodynamic effect from the flow field around the interacting droplet. The values of *E* obtained for the gravitational cases are 0.09 for the 15+20 μm droplets, 0.08 for the droplets of 10+20 μm and 0.05 for the 5+20 μm droplets. These values are well within the fairly broad range of other theoretical studies. Such a large range exists due to the necessary use of approximations and the lack of definitive experimental work for comparison.

Figure 2 shows how the collision efficiencies change with respect to the dissipation rates of the four numerical simulations. The efficiencies have been normalised by the corresponding gravitational values and all results show that the inclusion of turbulent accelerations in droplet motion acts to increase the efficiencies for all sizes and all flows considered. The largest increase in *E* is for collisions between the droplets of radius 5 and 20 μm. For this size combination the turbulent collision efficiency is almost three times the gravitational value. For the 10+20 μm case the collision efficiency increases from 1.7 times the gravitational case at the dissipation rate of about 100 cm^2 s^{-3} to 2.4 times greater for the most energetic flow field. The turbulent collision efficiency has the least change from the gravitational efficiency for the largest radius ratio, increasing from 1.15 to 2.4 times for the 15+20 μm droplets. Figure 1 shows that turbulence has the greatest effect on the geometric collision kernel for this size combination, however for the collision efficiency, figure 2 shows that turbulence has the least effect on these sizes. The reason for this is the small gravitational collision efficiency for the 5+20 μm droplets. *E* is determined by the relative velocities between interacting droplets and as table 1 shows $\langle|w_r|\rangle_n$ for the 5+20 and the 15+20 μm cases are comparable. The difference in the gravitational collision efficiencies for these radius ratios is much larger. Therefore when one compares the increase in the collision efficiency when the effects of turbulence are included, the small radius ratio has a greater sensitivity.

Turbulent Collision Kernels with Hydrodynamic Forces

Turbulent coalescence is governed by three processes: collision due to particle-turbulence interactions; collision efficiency due to particle-particle hydrodynamic interactions, and; coalescence efficiency as determined by surface sticking characteristics. Laboratory studies of colliding cloud droplets have shown that the coalescence efficiency is near unity [1]. Since detailed observations of the coalescence efficiency for small size droplets is inherently difficult to obtain in real clouds, previous theoretical studies have assumed that the collection efficiency is equal to the collision efficiency. Thus the problem of droplet growth through collection in a turbulent flow reduces to determining the collision rates that include hydrodynamic forcing.

Figure 3 shows the turbulent collision kernels that include the hydrodynamic forcing normalised by the corresponding gravitational kernel, that is

$$\left(E \times \Gamma\right)_{turb} / \left(E \times \Gamma\right)_{grav} \qquad (5)$$

In this figure each of the three normalised contributions in figure 2 have been multiplied together to give the total effect of the turbulence. For low dissipation rates turbulence has the most impact on the small radius ratios. For the 5+20 μm droplets at the dissipation rate of about 100 cm^2 s^{-3}, the increase in $E \times \Gamma$ is substantial at 3 times the gravitational value. This increases with increasing eddy dissipation rate to a value of more than 4 times the corresponding gravitational value. For the intermediate droplet pair of 10+20 μm, the increases range from almost 2 to 5 times the gravitational case across the four flow fields. The increase for the 15+20 μm droplets is quite modest for the least energetic flow, however it is for this droplet pair that we see the greatest effect of turbulence. The increase is greater than 9 times the gravitational case for the largest radius ratio in the most energetic flow.

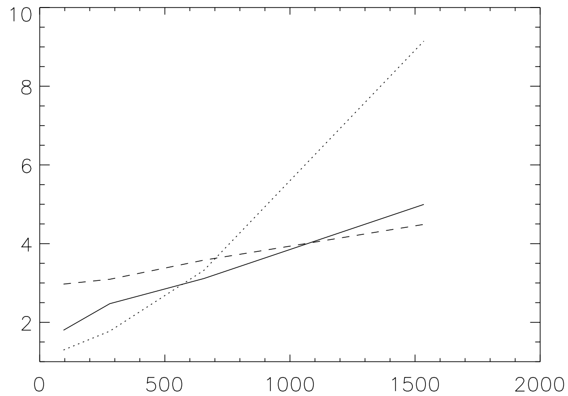

Figure 3. The turbulent collision kernel that includes the hydrodynamic forcing normalised by the corresponding gravitational kernel as a function of average eddy dissipation rate ($cm^2 s^{-3}$). The dotted line is the normalised collision kernel for the 15 and 20 μm droplets, the solid line is for the 10 and 20 μm droplets and the dashed line is for the 5 and 20 μm droplets.

Conclusions

The effect of turbulence on the collision rates of small cloud droplets has been explored by the use of a direct numerical simulation of the flow field. By implementing an efficient scheme to detect collisions, large numbers of droplets have been explicitly tracked as they move throughout the turbulent flow. The collision rates and characteristics of the droplets at the time of collision have been investigated. Four numerical simulations with increasing rates of eddy dissipation have been performed. As the intensity of the turbulence increases the deviation from gravitational statistics becomes greater. The turbulent geometric collision kernel that does not include the hydrodynamic forces between interacting droplets is greater than the corresponding gravitational kernel for each of the droplet size combinations considered. The turbulent collision kernel increases fairly linearly with increasing dissipation rate for each of the three size combinations. Turbulence has the most effect on the largest radius ratio, the 15+20 μm droplets. In this case the turbulent kernel is greater than the gravitational one by 1.1 times for the least energetic flow and 3.8 times for the most energetic flow. Analysis of the two kinematic quantities responsible for changes in the kernel shows that the increases in the geometric collision kernel for the small radius ratio droplets were predominately caused by increases in the relative radial velocities. For the 10+20 μm droplets, the increases in the kernel were caused by both increases in the relative radial velocities and the clustering of the droplets. The large increase for the 15+20 μm droplets was mostly due to the preferential concentration of the droplets, however, this size combination also had the largest increase in the relative radial velocities. This is because these droplets have the largest Stokes numbers and it has been shown that for the radial distribution function, the interaction between particles and turbulence is maximised when the Stokes number is of order 1.

The collision efficiency results presented are for the simplified problem of two interacting droplets, whereby the flow field is stationary over the time of interaction. Another assumption in the calculation of the collision efficiencies is that only those droplets that collide geometrically can collide when the hydrodynamic interactions are accounted for. For an initial separation distance of 0.04 cm, the collision efficiencies increased the least for the 15+20 μm droplets. For this case the collision efficiency increase ranged from 1.2 to 2.4 times the gravitational value across the four dissipation rates. The 10+20 μm case showed similar increases of 1.7 to 2.4 times the gravitational value and the collision efficiencies for the 5+20 μm droplets had the greatest increase of almost 3 times.

The overall turbulent coagulation process is governed by the product of the geometric collision kernel and the collision and coalescence efficiencies. Taking the coalescence efficiency to be 1, the increases that we have seen in both the geometric collision kernel and the collision efficiency translate into significant increases in the product of these two quantities. The lowest dissipation rates explored in this study have the greatest effect on the small radius ratio of more than 3 times the corresponding gravitational value. This is an encouraging result as it is at the beginning of the cloud lifetime when the dissipation rates are modest that collisions between small droplets will have the most effect on expediting the production of rain drops. The collision kernels that include the hydrodynamic forcing for the 5+20 and the 10+20 μm droplets increase linearly with dissipation rate from 3.0 and 1.8 times the gravitational values up to 4.5 and 5.0 times respectively. Low energy turbulence has the smallest influence on the collision kernel with hydrodynamic forces for the largest radius ratio. For this case the 15+20 μm droplets show increases of 1.3 and 1.8 times the gravitational case for the two lowest dissipation rates. However, the effect of turbulence is the most dramatic for this size combination, which for the most energetic flow has an increase of more than 9 times the gravitational value. These increases are significant and are expected to influence the evolution of the drop size distribution and accelerate the growth of rain drops.

Acknowledgments

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