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On Transient Flow in a Ventilated Filling Box

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Abstract

The transient flow driven by turbulent buoyant plumes rising from the floor of a ventilated filling box is examined. A rectangular box is considered and openings in the base and top of the box link the interior environment to a quiescent exterior environment of constant and uniform density. A theoretical model is developed for predicting the density stratification and the volume flow rate through the openings that lead to the known steady state. Predictions are compared with the results of small-scale analogue laboratory experiments in which saline solutions are used to create density differences in water tanks. The timescale for the flow to approach steady state depends on the relative magnitudes of two timescales; (t_f) proportional to the time taken for fluid from the plume(s) to fill a closed box and (t_d) the time taken to drain buoyant fluid from the ventilated box. Previous work has shown that the steady state is characterised by the dimensionless vent area A^*/H^2 and we re-interpret this quantity as the ratio of the timescales ($\mu = t_d/t_f$). For $\mu > \mu_c$ the depth of the buoyant upper layer is shown to exceed, or 'overshoot', the steady layer depth during the initial transient. Applications include the natural ventilation of buildings and some implications of our results to building ventilation are discussed.

Introduction

We consider the time-dependent stratification and flow induced by turbulent plumes in a ventilated box. Turbulent plumes in a sealed enclosure were examined by [1] in their 'filling box' paper. In a filling box, the plume rises to the top of the box and spreads laterally outwards to form a density interface between the plume outflow and the ambient fluid. This interface descends towards the plume source as ambient fluid is entrained into the plume and passes into the buoyant upper layer. [1] showed that the box fills with buoyant fluid on a timescale which depends on the floor area (S) and height (H) of the box, and the buoyancy flux (B) of the plume. This resulted in the 'filling time', t_f, of

$$t_f \propto \frac{S}{B^{\frac{1}{3}} H^{-\frac{2}{3}}}. \tag{1}$$

[8] investigated ventilated boxes, including the draining of buoyant fluid from a box via openings in the top (of area a_t) and base (of area a_b) connecting to a quiescent external ambient. In the absence of mixing the time taken to flush the box, the 'draining time', t_d, is given by

$$t_d \propto \frac{S}{A^*}\left(\frac{H}{g'}\right)^{\frac{1}{2}} \tag{2}$$

where $g' = g\Delta\rho/\rho$ is the reduced gravity of the buoyant layer, $\Delta\rho$ the density contrast between the layers, ρ the density of the external ambient and A^* the 'effective' area of upper and lower openings such that $a_b^* = 2^{\frac{1}{2}} c_b a_b$, $a_t^* = 2^{\frac{1}{2}} c_t a_t$ and

$$\frac{1}{A^{*2}} = \frac{1}{a_b^{*2}} + \frac{1}{a_t^{*2}} \tag{3}$$

where c_b and c_t denote the loss coefficients associated with flow through the respective openings. This draining time is valid for uni-directional flow through the openings which usually requires that a_b is greater than or roughly equal to a_t, see [4]. For high Reynolds number flows c_b and c_t are normally assumed constant (≈ 0.6).

[8] then consider a single continuous point source input of buoyancy at floor level and balance the draining flow rate with the supply flow rate from the plume and, thereby, examine 'emptying filling boxes', focussing on the steady states. A steady-state two-layer stratification is reached in which the upper buoyant layer drives a draining flow through the vents which is balanced by the filling flow from the plume. The steady upper layer depth ($H-h$) depends only on the vent area, the box height and the plume entrainment coefficient (α):

$$\frac{A^*}{H^2 C^{\frac{3}{2}}} = \frac{\zeta_{ss}^{\frac{5}{2}}}{\sqrt{1-\zeta_{ss}}} \tag{4}$$

where the subscript ss denotes 'steady state', $C = \pi(5/2\pi\alpha)^{\frac{1}{3}}(6\alpha/5)^{\frac{5}{3}}$ and $\zeta = h/H$. Herein we use $\alpha = 0.09$ which results in a value of $C = 0.16$. The steady interface height ζ_{ss} is thus a function of the box geometry only and the entire problem may be regarded as geometric.

In this paper we re-interpret the parameter $A^*/H^2C^{\frac{3}{2}}$ in (4), previously regarded as a dimensionless opening area, as the ratio of the two competing timescales t_d and t_f, and examine the influence of these two times on the transient development of the flow. We begin by developing a theoretical model of the transients and then compare this with the results of analogue laboratory experiments. We discuss these results in the context of ventilation of a lecture theatre. Conclusions are then drawn.

Mathematical model for the transient flow

Buoyancy is input from n equal non-interacting, localised point sources of buoyancy flux B at floor level ($z = 0$) in a rectangular box of height H and cross-sectional area S (independent of height). Openings, of area a_b and a_t, in the base and top of the box, respectively, connect the interior environment to a quiescent exterior environment of constant density ρ. Buoyancy transfers between the boundaries of the box and the fluid in the interior are assumed negligibly small.

The turbulent plumes rising from the sources entrain ambient fluid and on reaching the top of the box spread radially outwards. Following [1] the details of the outward motion are ignored and an infinitesimally thin buoyant layer forms at time $t = 0$ with an interface separating the layers. Over time the interface descends and increasingly buoyant fluid is fed into the layer. A pressure difference between the interior and exterior environments is thereby established which drives buoyant fluid out through the upper openings and draws in an equal volume of ambient fluid through the lower openings. Mixing between the inflowing fluid and the buoyant layer is assumed negligible and the flow through each opening is assumed to be unidirectional. As the layer deepens and increases in buoyancy, the volume flux out of the box increases and begins to balance the volume flux fed into the buoyant layer by the plumes. After a finite time, a steady-state flow is approached in which the volume fluxes balance, the level of the interface is constant and the stratification consists of two homogeneous layers as confirmed in the experiments of [8]. Although one might expect a transient density profile similar to that of a filling box, for the purposes of modelling the movement of the interface we assume that the buoyant layer is well mixed throughout the transients.

The time rate of change of the buoyant layer thickness ($H-h$) is governed by the difference between the volume flux of buoyant fluid supplied (nQ_p) via the n buoyant plumes, and the volume flux out through the upper openings (Q_{out}). Conservation of volume and buoyancy yield

$$\frac{dV}{dt} = nQ_p - Q_{out} \quad \text{and} \quad \frac{dVg'}{dt} = nB - B_{out} \tag{5}$$

respectively, where g' and $V = S(H-h)$ are the average buoyancy and volume of the buoyant layer. $B_{out} = g'Q_{out}$ is the flux of buoyancy out through the top openings. Introducing the non-dimensional interface height ζ and reduced gravity δ:

$$h = \zeta H \quad \text{and} \quad g' = \delta C^{-1} B^{\frac{2}{3}} H^{-\frac{5}{3}} \tag{6}$$

and using the plume flow rate scalings and the draining theory outlined by [8], the time rate of change of the buoyant layer depth and of the average layer buoyancy can be expressed as

$$\frac{d\zeta}{d\tau} = \frac{1}{\sqrt{\mu}}\sqrt{\delta(1-\zeta)} - \sqrt{\mu}\zeta^{\frac{5}{3}} \tag{7}$$

$$\frac{d\delta}{d\tau} = \sqrt{\mu}\frac{1-\zeta^{\frac{5}{3}}\delta}{1-\zeta} \tag{8}$$

where the non-dimensional timescale τ and parameter μ are

$$t = \tau\sqrt{t_d t_f} = \tau\left(\frac{S^2}{nC^{\frac{1}{2}}A^* B^{\frac{2}{3}} H^{-\frac{2}{3}}}\right)^{\frac{1}{2}} \tag{9}$$

$$\mu = \frac{t_d}{t_f} = \frac{nC^{\frac{3}{2}}H^2}{A^*} \tag{10}$$

and the timescales t_d and t_f are defined as

$$t_d = \frac{C^{\frac{1}{2}} S H^{\frac{4}{3}}}{A^* B^{\frac{1}{3}}} \equiv \frac{S}{A^*}\left(\frac{H}{g'_p|_{z=H}}\right)^{\frac{1}{2}} \tag{11}$$

$$t_f = \frac{S}{nCB^{\frac{1}{3}}H^{\frac{2}{3}}}. \tag{12}$$

g'_p denotes the reduced gravity of the plume. Further details of the model and generalisation to both point and line source plumes are given in [7]. The initial conditions considered are those of an initially empty box:

$$\delta = 1, \quad \zeta = 1 \quad \text{at} \quad \tau = 0. \tag{13}$$

The timescale t_d is the 'draining box' time and is proportional to the time taken for a buoyant layer of depth H and of buoyancy equal to that in the plumes at height H to drain completely through openings of effective area A^*. t_f is the 'filling box' timescale for n non-interacting plumes each of buoyancy flux B (see [1]). In other words, t_d relates to the draining of a ventilated box in the absence of a supply of buoyancy and t_f relates to the filling of an unventilated box supplied with a constant buoyancy flux from sources at floor level.

Theoretical Predictions

Solution of the governing equations (7) and (8) subject to (13) was achieved using a finite-differencing scheme. The key parameters examined are: the interface heights at maximum overshoot (ζ_{over}) and at steady state (ζ_{ss}), and the time taken to reach the maximum overshoot (τ_{over}) and the steady state (τ_{ss}). The time taken to reach the steady state is defined as the time taken for the ambient *layer* to reach 99% of its steady-state depth ($|\zeta-\zeta_{ss}| < 0.01$). When the interface overshoots the steady state, τ_{ss} is the time taken to overshoot and then settle back to 99% of the steady-state interface height. Numerical solutions for $10^{-3} < \mu < 10^5$ from $\tau = 0$ to $\tau = 50$ were evaluated; the finishing time chosen allowed the interface height to reach steady state for the full range of μ considered. The steady state is expected to be approached but never reached, however, given the (large) finish time of $\tau = 50$ the flow approached close enough to the steady state of (4) to be graphically indistinguishable.

Progress to steady state

The variation of the dimensionless volume flow rates, as supplied to ($\sqrt{\mu}\zeta^{\frac{5}{3}}$) and draining out of ($\sqrt{\delta(1-\zeta)}/\sqrt{\mu}$) the buoyant layer, leading to the steady state is shown in figure 1 for $\mu = 10$. A rapid decrease in the volume flux supplied to the layer by the plume is predicted as the layer descends towards the plume source. The volume flux draining out increases more gradually. The lines shown intersect at the point of maximum overshoot (see figure 2), after which the volume flux out exceeds the volume flux supplied to the layer and the layer thins as it approaches the steady state.

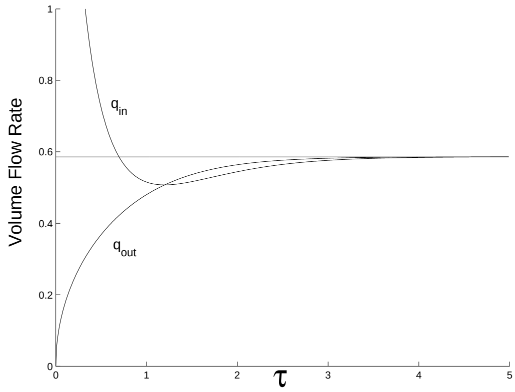

Figure 1: $q_{in} = \sqrt{\mu}\zeta^{\frac{5}{3}}$ and $q_{out} = \sqrt{\delta(1-\zeta)}/\sqrt{\mu}$ against τ for a point source plume and $\mu = 10$. The horizontal line represents the value of the volume fluxes at the predicted steady state (4).

Figure 2 shows profiles of ζ as a function of τ. An initial rapid deepening of the buoyant layer is predicted. For sufficiently large μ the buoyant layer depth exceeds (or overshoots) the steady value and then thins as it approaches steady state. The initial rate of change of the buoyant layer depth increases with increasing μ. The initial increase in layer depth results in the layer being supplied with increasingly buoyant fluid from the plumes, and thus a corresponding increase in the average buoyancy of the layer. The time taken to reach maximum overshoot can be seen to decrease as μ increases.

Amplitude of overshoot

For small values of μ no overshoot is predicted (see figure 2) but for larger values of μ the overshoot can be as high as 3.7% of H (this occurs at $\mu \approx 41.4$). Although the overshoot typically is only a small fraction of the box height, it can be a significant

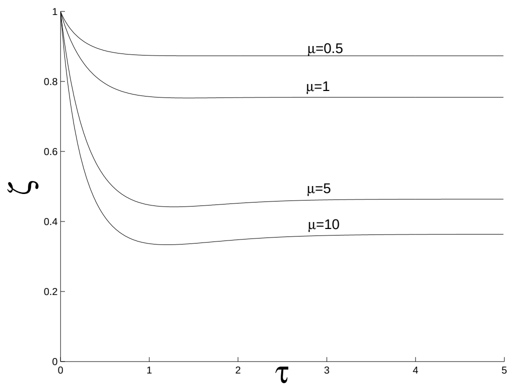

Figure 2: ζ vs. τ for point sources with $\mu = \{0.5, 1, 5, 10\}$.

fraction of the ambient lower layer depth when the lower layer is thin (i.e. for large μ).

Time taken to reach steady state

Figure 3 shows the time taken τ_{ss} to reach the steady-state layer depth. The dashed line is the time to reach ζ_{ss} the first time, i.e. as the interface descends. The solid line is the time taken to return to ζ_{ss} after overshooting, i.e. as the interface ascends. For $\mu < \mu_c$ there is no overshoot, and the layer merely increases to the steady layer depth. [7] predict that $\mu_c = 0.27$ and provide an explanation for the 'bulge' in the plot.

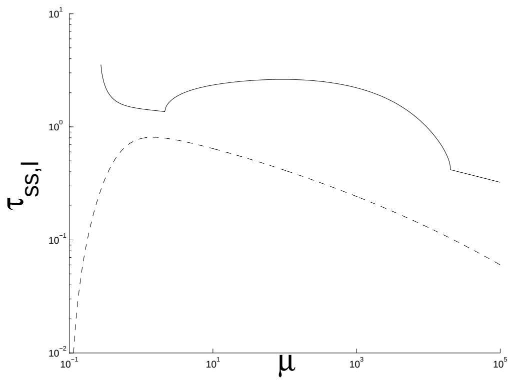

Figure 3: Time taken (τ_{ss}) to reach the steady layer depth during both the initial increase in layer depth (dashed line) and subsequent decay in layer depth towards the steady state (solid line).

Experiments

To test the validity of the modelling assumptions a series of small-scale laboratory experiments were performed in a large, clear-sided visualisation tank filled with fresh water. A clear Perspex box (cross section 0.5m×0.5m, height 0.2m) was immersed in the tank. Circular holes, of 3cm and 5cm diameter, in the top and bottom of the box (including a 1cm and a 0.5cm diameter hole in the bottom) could be opened or closed by removing or adding plugs. The range of μ possible was $0.1 < \mu < 35$, however, for $\mu > 10$ we were unable to reach the final steady state owing to the tank stratifying as saline solution flowed out of the box. Saline solution was supplied via constant head apparatus to a nozzle located in the centre of the box's top face. The (constant) volume flow rate of supply was finely controlled with a needle valve and measured using an in-line flowmeter. Dye added to the supply aided visualisation and the apparatus was diffusively back lit. The density of the ambient and supply was measured using an Anton Paar DMA 35N density meter (accuracy $5 \times 10^{-4}\mathrm{gcm}^{-3}$). The virtual origin of the plume source was located following [6]. An experiment was started by opening a tap supplying salt solution to the nozzle; the needle valve was pre-set so that the desired flow rate was achieved immediately. The upper opening area was typically a factor of two greater than the lower opening area in order to maintain a low inlet velocity. This low inlet velocity prevents the inflowing jets of fluid from disturbing the interface, and enabled a sharp interface to be maintained. However, due to the geometry of the box, for smaller values of μ (larger vent areas) we were unable to maintain this ratio resulting in disturbances on the interface. The digital image analysis system DigiFlow [5] was used to track the interface. A horizontal average of each time frame was taken (excluding the plume region), and the point of highest vertical gradient in the intensity (buoyancy) profile was taken as the interface height.

Results

A descending turbulent plume developed below the nozzle and after impinging with the base of the box spread radially outwards as a saline gravity current. This outflow is considered in an accompanying paper by the authors at the 15th AFMC. On reaching the box walls a well-defined saline layer, with depth approximately equal to the width of the plume at the base of the box, was clearly visible. The current sloshed up the side walls of the box before slumping back downward. This produced wave-like disturbances which propagated along the interface between the layers before dissipating over time. The initial slumping also produced mixing between the layers, however, at later times the only fluid crossing the density step was that entrained into the plume. Saline drained out through the base openings and was replaced by fluid of ambient density which flowed in through the top openings. For most μ the interfacial disturbances due to the flow through the openings were minimal and were wave-like with no mixing across the interface and, therefore, had no effect on the interface position. For small μ, however, the disturbances were more significant.

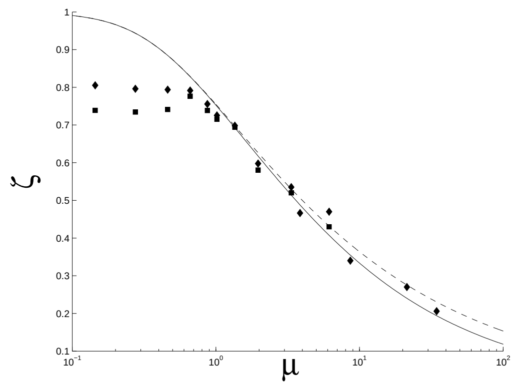

Figure 4: Predicted and measured values for ζ_{over} (diamonds and dashed line) and ζ_{ss} (squares and solid line).

During the initial transients the saline layer rapidly increased in depth (*cf.* figure 2) indicating that the draining rate was small compared with the supply rate to the layer from the plume. The subsequent development of the stratification within the box was observed to be dependent on the opening area:

(i) For relatively small openings ($\mu > 1$) the buoyant layer initially deepened and the interface became horizontal and sharper as the initial slumping phase decayed away and the mixed fluid at the interface was entrained into the plume. The layer depth increased to a maximum and then decreased to the steady-state depth. Other than the finite thickness of the initial outflow layer this is qualitatively in keeping with our model. Measurements of interface height show generally good agreement with theory for this range of μ (see figure 4). There is also good agreement between our theoretical predictions of τ_{over} and τ_{ss} for $\mu > 1$ (see figure 5) and our experiments.

(ii) For sufficiently large opening areas ($\mu < 1$) a different flow regime was observed. Instead of the hydrostatic two-layer stratification modelled, the interface in the region below the open vents was broken up and mixed by the inflowing jets of ambient fluid. Away from the openings the interface was also unstable with waves persisting. A significant initial overshoot of the interface height was also observed due to these disturbances. The final average interface height was not observed to vary with μ and was of the order of the plume width at $z = H$. Therefore, for $\mu < 1$, the experimental measurements of ζ and τ show poor agreement with our theoretical predictions.

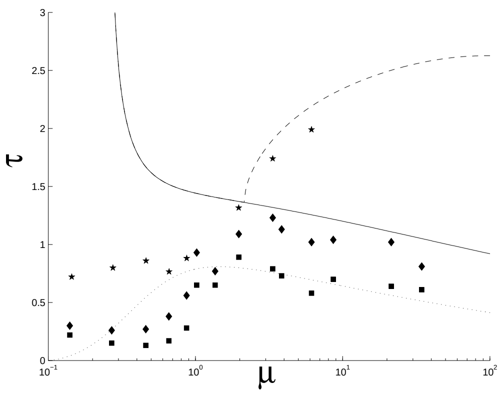

Figure 5: Predicted and measured values of τ_{ss} and τ_{over}. Squares and dotted line indicate the time taken to reach the steady-state height. Diamonds and solid line indicate the time taken to reach maximum overshoot τ_{over}. Stars and dashed line indicate the time taken to settle back to the steady state.

Discussion

Our model and experiments have shown that transient buoyancy-driven flow in a ventilated enclosure yields a reduced volume flow rate through the enclosure compared to the steady-state value and an overshoot of the buoyant upper layer for sufficiently large μ. We now briefly discuss the implications of these transient phenomena in the context of a lecture room.

We consider a lecture theatre with floor area 300m^2, height 6m and with 256 occupants. We assume that each occupant can be represented by a plume with a power output of 100W. We also assume an opening area of 1.5% of the floor area giving $A^* = 2.9\text{m}^2$. We model the heat input in two ways. Firstly as 256 individual plumes (one from each occupant) and secondly as a single plume (assuming they all merge near their source). The parameters used and resulting timescales are summarised in table 1. These values are representative of those used in ventilation models of actual buildings (see [9] & [3]).

The first point to note is the significant variation in μ depending on how the heat is input. Also the time taken in both cases to reach steady state is of order a quarter of an hour. This means that for the first quarter of a one hour lecture the ventilation rates will be less than designed for using a steady-state model.

	256 plumes	**single plume**
# Plumes	256	1
t_d	54	8.5
t_f	0.25	11
$\sqrt{t_d t_f}$	3.8	10
μ	201	0.8
τ_{over}	0.8	1.5
τ_{ss}	2.6	1.5
t_{over}(mins)	3	14
t_{ss} (mins)	10	14
Air changes/hr	6.6	3.9

Table 1: Typical lecture room parameter values and timescales for a floor area 300m^2. Times are given in minutes. Buoyancy flux has been calculated using $B = \frac{gP}{\rho T C_p}$ where P and T are the source power output (W) and temperature °K) [2], respectively.

Conclusions

Transient flow in ventilated boxes driven by a sudden increase in buoyancy flux has been investigated. The transients are governed by the relative magnitudes of the draining (t_d) and filling (t_f) timescales as characterised by the parameter $\mu = t_d/t_f$. The layer depth may overshoot the steady-state depth for $\mu > \mu_c$. Results of laboratory experiments show good agreement with our predictions for $\mu > 1$. However, below this value, a constant thickness layer was measured and found to be independent of μ.

Acknowledgements

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