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Outflow from a Plume Impinging on a Horizontal Boundary

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Abstract

The radial outflow from a Boussinesq turbulent plume impinging on a horizontal boundary is examined both theoretically and experimentally. We gain insight into the transition from the vertical plume flow to the horizontal gravity current flow by drawing an analogy between classifying a constant buoyancy flux radial gravity current in terms of its source conditions and forced, pure and lazy plumes. Using dimensional arguments and existing experimental results for pure gravity currents and plumes, we demonstrate that the outflow will be initially jet like, before adjusting into a gravity current. This finding is supported by the results of our experiments.

Introduction

We consider the collision of an axisymmetric turbulent plume flow with steady source conditions and a horizontal boundary in a quiescent environment. In practice these flows are relatively common, for example, a fire plume or thermal plume from a heat source in a room impinging on a ceiling (in this case a solid boundary), or a buoyant waste water plume reaching the surface of a sea into which it is pumped. In this work we restrict our attention to situations where the density difference between the plume and the surroundings is small compared with the density of the surroundings.

On impinging with the boundary the flow from the plume will spread horizontally outwards and in the far field develop into a constant flux radial gravity current. Near the point of impact, however, there is an adjustment region over which the flow develops into a pure gravity current - here we use ‘pure’ to refer to a current with a constant Froude number.

The details of this adjustment process and the radial extent over which this adjustment occurs are not well understood and form the motivation for the present study. Knowledge of the process of transition from vertical plume flow to horizontal gravity current flow is key to understanding a number of flows of practical interest. For example, when a thermal plume impinges on a ceiling in a confined room it spreads out and hits the side walls. In order to understand how far down the wall the heat or smoke will then initially descend requires an understanding of the dynamics of the initial outflow. Though not covered here, this problem is currently under investigation by the authors.

A schematic of the flow considered herein and the nomenclature adopted is given in figure 1. We denote the plume buoyancy flux by F, the vertical distance from the plume to the boundary by H, the radial coordinate by r and the thickness of the gravity current by $\Delta(r)$. We begin by stating a key result relating to the spread of constant flux radial gravity currents before considering the initial adjustment region. We then present a mathematical model for the bulk properties of the flow and compare our predictions to experimental measurements of the movement of the gravity current front.

In [3] a balance balance between the buoyancy force $(\rho g'\Delta^2)$ and inertial force $(\rho r^2\Delta/t^2)$ was used to establish a relationship for the position of the front of the gravity current with time for the inertial regime:

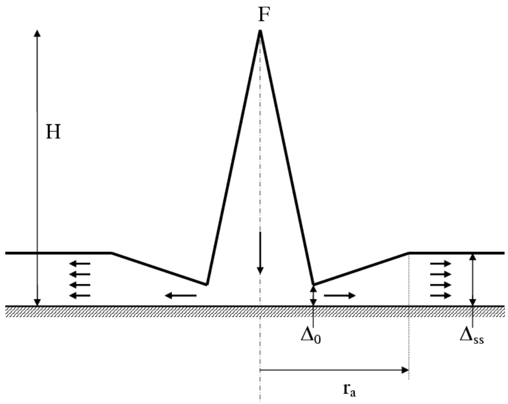

Figure 1: Schematic of the outflow from a plume impinging on a horizontal boundary showing the nomenclature adopted.

$$r = c_i F^{1/4} t^{3/4}. \tag{1}$$

Following on from this work [2] solve the governing equations and obtain the same time scalings as [3]. Experimental results suggest $c_i = 0.84$, hence

$$r = 0.84 F^{1/4} t^{3/4}. \tag{2}$$

This equation for the front movement implies a constant Froude number for the gravity current. If the ‘source’ conditions which characterise the supply of material to the current do not result in an identical Froude number, as will in general be the case, then there will be an adjustment region (in the near field) before the flow becomes a pure constant buoyancy flux radial gravity current (in the far field).

In this paper we draw an analogy between classifying radial gravity currents and classifying plumes in terms of their respective source conditions. We demonstrate that the adjustment region for the radial flow may be regarded as similar to either forced plumes [8] or lazy plumes [5] which themselves adjust to a pure plume in the far field. We also show that for a plume impinging on a horizontal boundary, the flow in the adjustment region is that of a forced radial gravity current regardless of the distance between source and boundary.

Source Conditions and Length Scales

For a constant buoyancy flux radial gravity current the main parameters are the thickness ($[\Delta] = L$), radial distance ($[r] = L$), and buoyancy flux ($[F] = L^4T^{-3}$). The other parameters of note are the volume flux ($[Q] = L^3T^{-1}$) and momentum flux ($[M] = L^4T^{-2}$). Assuming that for the fully-developed flow the current has a constant thickness and velocity (v) we can use dimensional analysis to develop scalings for the volume flux ($2\pi r\Delta v$) and momentum flux ($2\pi r\Delta v^2$) in terms of the thickness, radius and buoyancy flux:

$$Q \sim \Delta F^{1/3} r^{2/3} \quad \text{and} \quad M \sim \Delta F^{2/3} r^{1/3}. \tag{3}$$

The velocity of the current

$$v = \frac{dr}{dt} \sim F^{1/3} r^{-1/3}. \quad (4)$$

It is possible to derive the latter result by differentiating (2):

$$v = \frac{dr}{dt} = \frac{3 \times 0.84 F^{1/4}}{4t^{-1/4}} = 0.59 F^{1/3} r^{-1/3} \quad (5)$$

which can now be used to estimate the values of the constants in (3):

$$Q \approx 3.7 \Delta F^{1/3} r^{2/3} \quad \text{and} \quad M \approx 2.2 \Delta F^{2/3} r^{1/3}. \quad (6)$$

These equations give straightforward estimates of the volume and momentum flux at any given radial distance, but for chosen values of the fluxes Q and M they can also be inverted to provide an estimate of the radial distance a gravity current of buoyancy flux F would need to travel in order to obtain those flux values. It is in this sense that we define two radial length scales, the volume flux length scale (r_Q) and the momentum length scale (r_M):

$$r_Q = 3.7^{-3/2} \Delta_0^{-3/2} Q_0^{3/2} F^{-1/2} \quad (7)$$

$$r_M = 2.2^{-3} \Delta_0^{-3} M_0^3 F^{-2} \quad (8)$$

where the subscript 0 indicates the initial value. Note that the plume and current source buoyancy fluxes are identical as the environment is unstratified, thus $F = F_0 = constant$. Drawing an analogy with plume theory (see [5]) the gravity current can be regarded as 'forced' if $r_M > r_Q$, 'lazy' if $r_M < r_Q$ and 'pure' if $r_M = r_Q$. We can also define a ratio of these length scales analogous to that for plumes:

$$\Gamma = \frac{r_Q}{r_M} = 1.5 \frac{Q_0^{3/2} \Delta_0^{3/2} F^{3/2}}{M_0^3}. \quad (9)$$

Again the flow in the adjustment region will be 'forced' for $\Gamma < 1$, 'pure' for $\Gamma = 1$ and 'lazy' for $\Gamma > 1$.

For the case of a highly 'forced' gravity current one would expect a development region that behaved like a radial jet. For a smooth boundary this is a constant momentum flux flow with a linear growth in current thickness with radius. This linear radial growth is observed in the work of [7] and [6]. It is therefore a reasonable approximation that the transition region is 'forced' and will behave as a radial jet with a linear growth in thickness with radius and constant momentum flux (see, for example, [1]).

The 'source' conditions for the radial flow are those supplied by the pure plume of buoyancy flux (F). Assuming top-hat profiles in the plume, the radius (b) and volume flux (Q_p) and momentum flux (M_P) at the boundary can be expressed as:

$$b = \frac{6\alpha_t H}{5} = 0.15H \quad (10)$$

$$Q_p = \pi \left(\frac{5F}{8\pi\alpha_t}\right)^{1/3} \left(\frac{6\alpha_t H}{5}\right)^{5/3} = 0.16 F^{1/3} H^{5/3} \quad (11)$$

$$M_p = \pi \left(\frac{5F}{8\pi\alpha_t}\right)^{2/3} \left(\frac{6\alpha_t H}{5}\right)^{4/3} = 0.35 F^{2/3} H^{4/3} \quad (12)$$

where $\alpha_t \approx 0.09\sqrt{2}$ (see [5]) is the plume entrainment coefficient appropriate for top-hat profiles. We now use (10) as the initial gravity current radius and using continuity we take (11) as the initial volume flux. However, it is likely that there will be losses as the plume impinges on the solid boundary. We therefore assume that the initial gravity current momentum flux is given by γM_p, for some constant γ. We can now establish the boundary conditions on the radial outflow, namely,

$$r_0 = 0.15H, \quad Q_0 = 0.16 F^{1/3} H^{5/3} \quad (13)$$

and

$$M_0 = \gamma M_p = 0.35 \gamma F^{2/3} H^{4/3}. \quad (14)$$

Therefore, for the initial outflow thickness we get

$$\Delta_0 = \frac{Q_0^2}{2\pi r_0 M_0} = \frac{0.068}{\gamma} H. \quad (15)$$

This implies that the initial outflow thickness increases as the energy loss (due to the change in flow direction) increases.

The next step is to evaluate the length scales defined in (7) and (8) using (15). This leads to

$$r_Q = 0.41 \gamma^{3/2} H \quad \text{and} \quad r_M = 7.1 \gamma^6 H, \quad (16)$$

or

$$\Gamma = 0.58 \gamma^{-9/2}. \quad (17)$$

We note the particularly sensitive dependence of r_M and Γ on γ and that (17) indicates that the flow is forced near the point of impingement provided $\gamma > 0.53$. It is expected that the losses will not be this significant and, therefore, we choose to model the transitional flow as a forced radial jet.

We will assume that the flow behaves as a turbulent radial momentum jet near the point of impingement with no loss in momentum flux as the flow spreads across the boundary. It it therefore expected to entrain ambient fluid and increase linearly in depth with radius [1]. This will increase the local value of Γ until it reaches the pure gravity current balance of $\Gamma = 1$. The flow will then behave as a radial gravity current as given by (2). For a momentum jet we know that:

$$\Delta = \frac{d\Delta}{dr}(r - r_0) + \Delta_0 \quad \text{and} \quad M = M_0 = \gamma 0.35 F^{2/3} H^{4/3} \quad (18)$$

where $d\Delta/dr$ and γ are our unknown constants, and r_0 and Δ_0 are given by (13) and (15), respectively. Carrying these unknowns through we can write expressions for the velocity v and volume flux Q:

$$v = \frac{\gamma^{1/2}\sqrt{0.33} F^{1/3} H^{2/3}}{\sqrt{2\pi r \Delta}} \quad \text{and} \quad Q = \sqrt{2\pi r \Delta} \gamma^{1/2} \sqrt{0.33} F^{1/3} H^{2/3}. \quad (19)$$

To leading order in r the outflow thickness is given by $\Delta \approx \frac{d\Delta}{dr} r$, hence, using (19) we estimate the movement of the front with time in the adjustment region as

$$\frac{dr}{dt} \approx \sqrt{\frac{0.35}{2\pi}} \sqrt{\frac{\gamma}{\frac{d\Delta}{dr}}} \frac{F^{1/3} H^{2/3}}{r}. \quad (20)$$

This leads to

$$r \approx 0.69 \left(\frac{\gamma}{\frac{d\Delta}{dr}}\right)^{1/4} F^{1/6} H^{1/3} t^{1/2} \quad (21)$$

to leading order in r.

We now introduce length and time scales based on the vertical separation H and the plume buoyancy flux F. Dimensionless time and radial distance, respectively, are given by

$$\tau = \frac{t}{H^{4/3} F^{-1/3}} \quad \text{and} \quad \phi = \frac{r}{H}. \quad (22)$$

This allows us to write expressions for the front movement with time in non-dimensional terms. In the initial forced adjustment region we get from (21):

$$\phi = 0.69 \left(\frac{\gamma}{\frac{d\Delta}{dr}} \right)^{1/4} \tau^{1/2} \tag{23}$$

and for the pure gravity current flow described by [2] we get

$$\phi = 0.84\tau^{3/4}. \tag{24}$$

In order to establish the value of the constant in (23) and the radial extent of the forced region we now turn to laboratory experiments.

Experiments

A series of laboratory experiments were performed to examine the plume outflow. A negatively buoyant turbulent plume was created by injecting a salt solution (NaCl) at constant volume flow rate into a large visualisation tank filled with fresh water. The plume descended and impinged on a horizontal false bottom in the tank. As the radial flow from the plume spread out it was filmed using a CCD digital camera and the captured images processed using the software DigiFlow [4]. A schematic of the experimental setup is shown in figure 2.

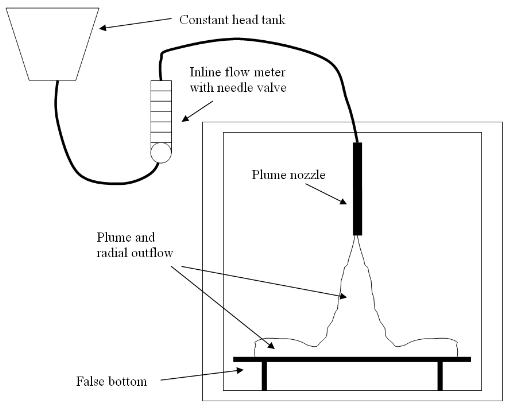

Figure 2: Schematic of experimental setup for tracking the front movement and thickness of the plume outflow.

Jet region

The first set of measurements reported is the dependence of the adjustment radius (r_a), see figure 1, on the separation distance (H) for fixed plume source conditions. For these experiments the plume was dyed with Sodium Fluorescein and a thin vertical light sheet was projected up from below the tank. Once the outflow was fully established a time-averaged image (see figure 3 for a typical image) of the flow was taken and the radial distance (r_a) at which the outflow was thickest measured . The separation H was varied from 5.5cm to 18.2cm. A plot of r_a against H is shown in figure 4. The line plotted is based on a best fit of the data forced through the origin and is given by

$$r_a = 0.66H. \tag{25}$$

Front movement

Using the same experimental setup a series of experiments was run to measure the front position of the outflow as a function of time. In these experiments the apparatus was back lit and the

Figure 3: Time-averaged image of the outflow showing the radial growth in thickness near the source and adjustment to a constant thickness gravity current. The right-hand boundary of the image is the plume centreline. The outflow is the horizontal dark grey region.

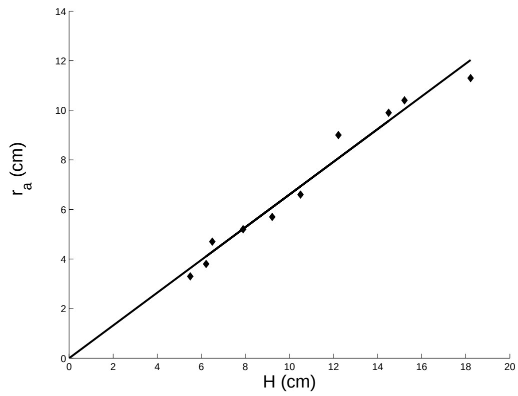

Figure 4: Plot of the radius r_a at which the outflow has maximum thickness as a function of the plume-boundary separation H. The line is given by $r_a = 0.66H$.

plume was dyed using a blue food colouring. A series of typical photos from these experiments is shown in figure 5.

A series of 5 experiments was performed, the plume-boundary separation and buoyancy flux are given in table 1. The plume was activated and the front movement was filmed. After each experiment was completed the film was reviewed and measurements of the front position against time were recorded. The time origin was taken as the moment the plume came into contact with the horizontal boundary. Measurements of front movement against time are plotted in figure 6.

Using the scalings given in (22) we can collapse all the data from figure 6 onto a single line. This scaled front movement is shown in figure 7. Based on our analysis we expect that for small ϕ there would be a jet-like region where the front move-

Experiment	H(cm)	$F(\mathrm{cm}^4\mathrm{s}^{-3})$	Symbol
A	47.5	255	$+$
B	50.5	255	$\times$
C	21.5	255	$*$
D	32.3	182	$\star$
E	7.1	122	$\circ$

Table 1: Experiment parameters for the five profiles presented in figures 6 and 7.

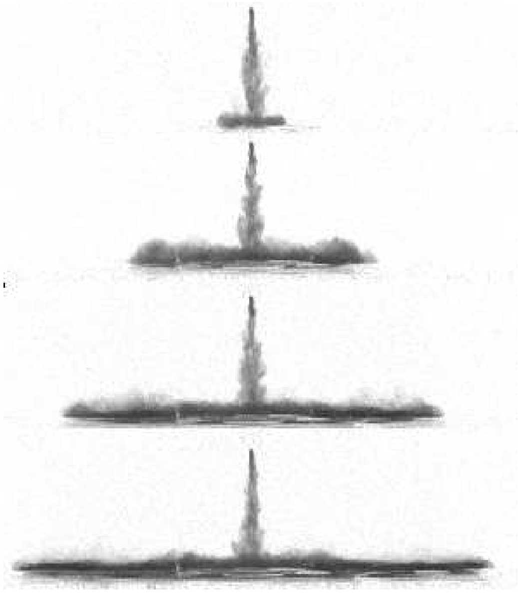

Figure 5: Series of digital images showing the descending plume and the horizontal outflow at various times during an experiment.

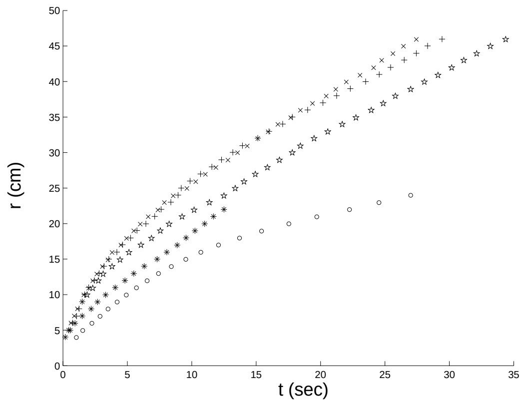

Figure 6: Plot of the front position r (cm) against time t (sec) for five experiments. + experiment A, $\times$ experiment B, $*$ experiment C, $\star$ experiment D and $\circ$ experiment E.

ment scaled on $\tau^{1/2}$ followed by a pure gravity current region where $\phi \sim \tau^{3/4}$. This behaviour is clearly seen in figure 7

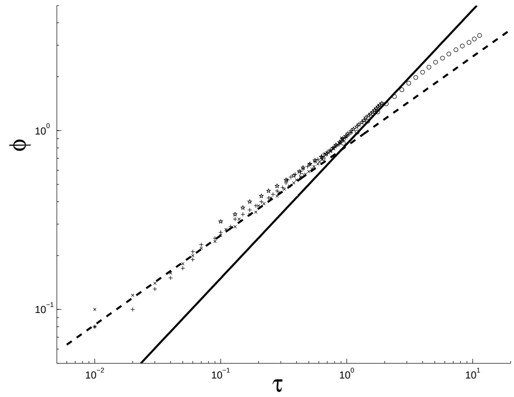

Figure 7: Non-dimensional front position ϕ against time τ showing the collapse of the experimental data and the 'forced' jet-like region near the point of impingement. The dashed line is given by $\phi = 0.85\tau^{1/2}$, *cf.* (23). The solid line is given by $\phi = 0.84\tau^{3/4}$. The discrepancy between the data and theory for large ϕ is due to the increasing influence of viscous forces.

Conclusions

We have demonstrated that the outflow from a turbulent plume that impinges on a horizontal boundary behaves as a radial forced jet near the point of impingement. Downstream of this initially forced region the flow behaves as a constant buoyancy flux radial gravity current. Results of our experiments demonstrate that the adjustment region scales linearly on the vertical distance of the plume source from the horizontal boundary (see figure 4). We have also demonstrated that the dimensionless front position $\phi = r/H$ can be described in terms of the non-dimensional time $\tau = t/(H^{4/3}F^{-1/3})$ by (23) and (24). Experimental results have shown that the full description of the front movement with time is given by:

$$\phi = \begin{cases} 0.85\tau^{1/2} & \phi < 0.66 \\ 0.84\tau^{3/4} & \phi \geq 0.66. \end{cases} \tag{26}$$

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