15th Australasian Fluid Mechanics Conference
The University of Sydney, Sydney, Australia
13-17 December 2004

The Cut-Off Frequency of Constant Temperature Hot-Wire Systems in Turbulent Velocity Measurements

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Abstract

The analytical solution for the frequency response of a constant temperature hot-wire system at given turbulent velocity fluctuations are used to investigate the effects on the cut-off frequency of the hot-wire system under different operating conditions. It is found that the optimal frequency response of the hot wire system determined by the traditional square wave test cannot guarantee the stability of the hot wire system under velocity perturbations. It is also found that the cut-off frequency depends not only on the hot-wire length to diameter ratio, the mean velocity, the hot-wire materials, and the overheat ratio as previously found, but also on the physical wire length, the resistances used in the Wheatstone bridge, and the gain of the amplifier in the feedback loop. Hot- wire systems with preferred operating conditions are recommended.

Introduction

Hot wires have been used for measuring velocities of turbulent gas flows for many years. Recently, Li [1] has for the first time derived an analytical solution for temperature fluctuations along a single hot wire when it is exposed to turbulent velocity fluctuations of different frequencies and wave numbers. The solutions show that attenuation to the velocity fluctuations exists at high frequencies and depends on the relative end-conduction heat loss from the hot wire to its supports. This result is consistent with the results obtained by Freymuth [2] based on intuitive argument and the numerical results of Morris and Foss [3] for a specific hot wire.

Li [1] used a simplified circuit for a hot-wire anemometer to analyse the frequency response of the combined hot wire and its anemometer with specific electronic components. The frequency response was investigated for given velocity fluctuations rather than introducing voltage perturbations as that normally performed during experiments through a square wave test. As Khoo et al. [4] pointed out, from a user's point of view, the cut-off frequency determined from the velocity fluctuation serves as a more accurate indicator of the overall frequency response of the hot-wire system. Li [1] found that even though the frequency response of the anemometer (as that determined from a square/sine wave test) can be very high, the cut-off frequency of the combined hot wire and its anemometer (it is called the hot-wire system here) can be low, and depends on the hot-wire length to diameter ratio, Reynolds numbers, overheat ratio, and the hot-wire material used. The analytical solutions were compared with the experimental results of Khoo et al. [4], who measured the frequency response of a hot-wire system by introducing velocity fluctuations to a marginally elevated and flush-mounted 5 μm tungsten wire of 1.2 mm long. Li [1] found that the theoretical results agree remarkably well with the experimental results of Khoo et al. [4] at the similar operating conditions.

In turbulence measurements using hot wires, the wire length is in general kept at $2l/d > 200$ following the recommendation of Champagne et al. [5]. Here $2l$ is the length of the hot wire and d is the diameter of the hot wire. On the other hand, it has also been common experience that hot wires in general cannot resolve all the turbulence length scales when the Reynolds number is high. The reason for this is that, at high Reynolds numbers, the hot wire used is longer than the small length scales of the turbulence, and it can only measure an averaged turbulent velocity along the wire. In the past, it has always been assumed that the spatial resolution problem of the hot wire depends only on the length of the wire because of the falsely high cut-off frequency determined from the square wave test. In light of the results of Khoo et al. [4] and Li [1], this may not be the case.

In this paper, we will investigate the effects from using different electronic components in the hot-wire anemometer and hot-wire diameters on the cut-off frequency of the hot-wire system. For a given hot wire in a specific flow field, it will be shown that the electronic components used in the Wheatstone bridge and the feedback circuit in the anemometer can all affect the cut-off frequency of the hot-wire system, even though their effect on the cut-off frequency of the anemometer itself is normally small. It is also found that not only the cut-off frequency of the hot-wire system is in general much lower than that determined from the square wave test but also an optimally tuned frequency response using square wave test cannot always guarantee the stability of the system when performing measurements. Recommendations will be made on the operating conditions of the hot wire with anemometer using preferred electronic components.

The hot-wire system and the theoretical solutions

In Li [1], the hot wire was assumed to be in cylindrical shape and is operated with an overheat ratio *a*, the ends of the hot wire are connected to the prongs (no stubs), and the hot wire is operated at constant temperature mode with the ambient temperature kept constant.

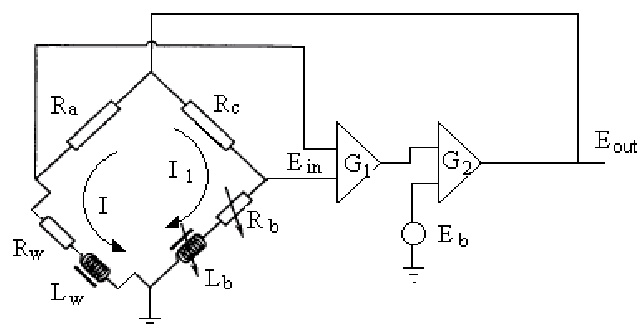

Figure 1. Schematic diagram of a constant temperature hot wire anemometer (CTHWA)

Figure 1 shows a schematic diagram of a Constant Temperature Hot-Wire Anemometer (CTHWA). In the figure, R_a and R_c are the two resistors in the upper arms of the Wheatstone bridge, R_w the hot wire resistance at the operating temperature, L_w the inductance from the hot wire cable, R_b and L_b the adjustable resistor and inductor for bridge balance, I and I_1 the electrical currents in the Wheatstone bridge, G_1 and G_2 the gains of the two stage amplifiers, E_{in} and E_{out} the instantaneous voltages at the input and output of the amplifiers, and E_b is a mean voltage for offset.

The solution for the frequency response of the hot wire system including the hot wire and its anemometer, as according to Li [1], is

$$\frac{\tilde{e}_{out}'}{\tilde{u}_1'} = \frac{A}{B - CP} \tag{1}$$

where

$$\tilde{e}_{out}' = \frac{\tilde{e}_{out}}{\overline{E}_{out}}, \qquad \tilde{u}_1' = \frac{\tilde{u}_1}{U}$$

$$A = \frac{\chi \overline{Q}}{Y_s^2 Y^2} \frac{\sin(\beta)}{\beta}[1 - f(Y) - f(Y_s) + g(Y) + g(-Y)]$$

$$f(x) = \frac{\beta}{x^2 + \beta^2}\left(\frac{x \tanh(x)}{\tan(\beta)} + \beta\right)$$

$$g(x) = \frac{\beta}{2[(Y_s + x)^2 + \beta^2]}\left[\frac{(Y_s + x)(\tanh(Y_s) + \tanh(x))}{\tan(\beta)}\right.$$
$$\left. + \beta(1 + \tanh(Y_s)\tanh(x))\right]$$

$$P = \frac{2\overline{Q}}{Y_s^2}\left[\frac{\tanh(Y_s)}{Y_s} - 1\right] +$$
$$\frac{2\overline{Q}^2}{\tau s}\left[\frac{\tanh(Y_s)}{Y_s^3} - \frac{\tanh(Y)}{Y^3} + \frac{\tau s}{Y_s^2 Y^2}\right]$$

$$B = \frac{[(M'' s^2 + M' s + 1)\sum Z - K Z_0](R_a + R_w)}{K R_a R_0 (R_c + Z_b)}$$

$$C = \frac{R_a + R_w}{R_a + Z_w}\left[1 + \frac{(M'' s^2 + M' s + 1)\sum Z - K Z_0}{K R_a (R_c + Z_b)}\right]$$

$$Q = \frac{4 I^2 R_{0x} \alpha}{\pi k_w}\left(\frac{l}{d}\right)^2, \qquad \xi = 4 Nu \frac{k_f}{k_w}\left(\frac{l}{d}\right)^2$$

$$Y = \sqrt{\xi - \overline{Q}}, \qquad Y_s = \sqrt{\xi - \overline{Q} + \tau s}$$

$$\tau = \rho_w C_w \frac{l^2}{k_w}, \; K = G_1 G_2, \; \chi = -\frac{U}{Nu}\frac{dNu}{dU}\overline{\xi}$$

$$\sum Z = (R_a + Z_w)(R_c + Z_b), \; Z_0 = R_a Z_b - R_c Z_w$$

$$Z_w = R_w + L_w s, \qquad Z_b = R_b + L_b s$$

Here s is the Laplace variable, ~ means the Laplace transform, M'' and M' are the time constants of the feedback circuit as given by Freymuth [6] and they are connected to the Gain Bandwidth Product (GBP) of the amplifiers, u_1 the time variation of the velocity fluctuation, U the mean velocity, Nu the Nusselt number, Re_w the Reynolds number based on the hot-wire diameter, k_w and k_f the thermal conductivities of the hot wire and the fluid respectively, R_0 and R_{0x} the electrical resistance of the hot wire and that of per unit length at ambient temperature respectively, α the thermal coefficient of electrical resistivity of the hot wire, $\beta = kl$ the non-dimensional wave number of the velocity fluctuations along the hot wire, and over-bar means steady condition. For detailed derivation of these relationships, please refer [1].

Using equation (1), Li [1] shows that the cut-off frequency f_c of the hot-wire system with $\beta = 0$ increases with increasing $2l/d$, U, and a, and with decreasing k_w. Here frequency response is defined as the ratio between the square of the voltage fluctuations at the anemometer to the square of the velocity fluctuations at the hot wire at each frequency, and f_c is defined as the frequency where the frequency response has dropped by *3dB*. It was found that this cut-off frequency is related to the relative heat loss from the end conduction.

The cut-off frequency f_c of the hot-wire system

Figure 2 shows the frequency response of the hot wire system of three wires in decibels: *5µm* tungsten wire, *2.5µm* tungsten wire and *2.5µm* Pt/Ir (90/10) wire all at $2l/d = 200$, $a = 0.5$, and $U = 10\ m/s$. In calculating the results, $R_a = 100\ \Omega$, $R_c = 1000\ \Omega$, $K = 1000$, $L_w = 15\mu H$, $GBP = 5\times10^7$, the bridge was tuned with $R_b = 9.98R_w$, $L_b = \lambda L_w R_b/R_w$, and $\lambda = 1.00285\text{-}1.0026$ to achieve an optimal frequency for the CTHWA with an approximate14.6% overshot as that specified by Brunn [7]. The slight off balance of the Wheatstone bridge with $R_b = 9.98R_w$ is consistent with the recommendation of Perry [8] and in practice this is achieved by adjusting the offset voltage E_b.

Figure 2 also shows the frequency response of the CTHWA and the attenuation of the hot wire to the velocity fluctuations for the *5µm* tungsten wire. The overshot of the CTHWA can be clearly seen and the attenuation of the hot wire to the velocity fluctuation approaches a lower plateau at high frequency as that found in [1-3]. The cut-off frequencies of the CTHWA for the 2.5µm wires are slightly higher than that given in the figure and are not shown for clarity. The cut-off frequency of the CTHWA determined from figure 2 is *900kHz*. This is also the cut-off frequency as determined from the impulse response using MATLAB, and that would be determined from the square wave test in practice. This high cut-off frequency for the CTHWA is because of the high *GBP* values used for the amplifiers and the circuit in figure 1 is an idealization of the actual hot-wire anemometer.

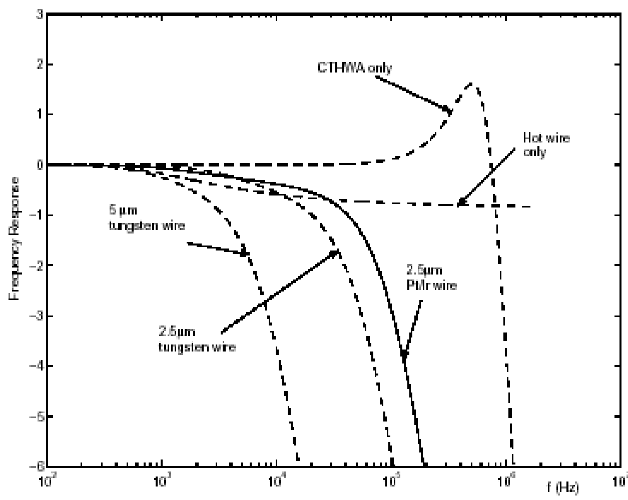

Figure 2 Frequency responses of hot wire and its anemometer

Figure 2 shows that the cut-off frequencies of the hot wire system are *8.2kHz* for the *5μm* tungsten wire, *52kHz* for the *2.5μm* tungsten wire and *100kHz* for the *2.5μm* Pt/Ir wire. These frequencies are much lower than that from the CTHWA alone, and depend strongly on the physical wire length and the wire materials. The time constants of the three wires are $\tau = 3.8\ ms$ for the *5μm* tungsten wire, *0.96ms* for the *2.5μm* tungsten wire and *5.6 ms* for the *2.5μm* Pt/Ir wire, respectively. The reason that the cut-off frequency of the *2.5μm* Pt/Ir wire is higher than that of the tungsten wire at the same diameter is because the thermal conductivity of the Pt/Ir wire is 31 W/mK while that of the tungsten is 174 W/mK even though the time constant of the former is much larger than the later.

Figure 2 shows that the cut-off frequency of the *2.5μm* wires are much higher than that of the *5μm* wire, and that of the *2.5μm* Pt/Ir wire is higher than that of the *2.5μm* tungsten wire. Thus it can be concluded that *2.5μm* Pt/Ir wire should be the preferred wire to use in turbulent measurements.

Figure 3 shows the effect of the amplifier *GBP* on the cut-off frequencies of the CTHWA and the hot-wire system for a *2.5μm* Pt/Ir wire. In figure 3, $GBP = 10^6$, 10^7 and 5×10^7 are used with $\lambda = 1.042,\ 1.0049$ and 1.0026, respectively, for optimal frequency response of the CTHWA. The other parameters are the same as those used in figure 2. It can be seen from figure 3 that the cut-off frequencies of the CTHWA are *43 kHz*, *380 kHz* and *1.1 MHz*, while those of the hot-wire system are *60kHz*, *240kHz* and *100kHz*, respectively, with $GBP = 10^6$, 10^7 and 5×10^7. However, it can be seen from figure 3 that the frequency response of the hot-wire system is unstable at $GBP = 10^6$ and thus should not be used even though the cut-off frequency of the hot-wire system is higher than that of the CTHWA. This also shows that the hot-wire anemometers tuned for optimal frequency response using the square wave test may not guarantee the stability of the hot-wire system. At $GBP = 10^7$, the cut-off frequency of the how-wire system is close to that of the CTHWA and higher than that at $GBP = 5\times10^7$. However, a close look at the results show that the frequency response of the hot-wire system with $GBP = 10^7$ is close to loss its stability. With imperfections in the electronic components and at higher mean velocities, it is expected that the hot-wire system will loss its stability for a *2.5μm* Pt/Ir wire with $GBP = 10^7$. Because of this, it is recommended that $GBP > 10^7$ should be used. However, hot-wire anemometers with $GBP = 10^7$ may be used for *5μm* tungsten wires since the cut-off frequency for this wire is well below the cut-off frequency of the CTHWA at such GBP.

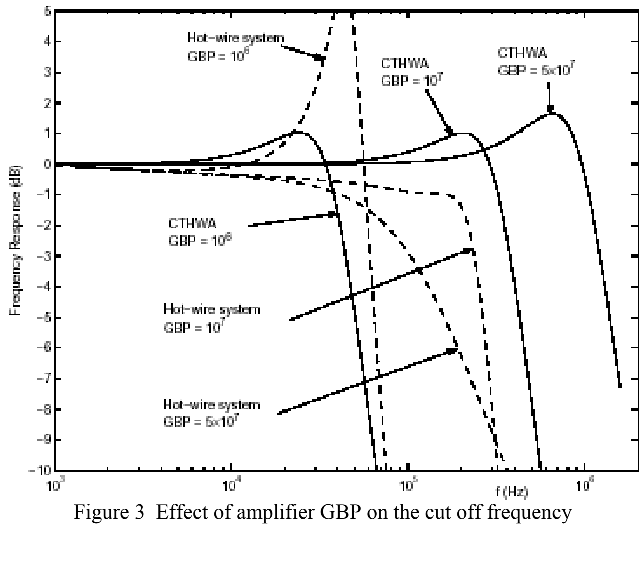

Figure 3 Effect of amplifier GBP on the cut off frequency

Figure 4 shows the effect of the feedback gain *K* on the cut-off frequency of the hot wire system. In the present analysis and that of [1], the two-stage amplifiers have been assumed to have the same gain, ie. $G_1 = G_2$. The hot wire used is a *5μm* tungsten wire. Again, L_b has been adjusted to have the optimal frequency response for the CTHWA. The other parameters used are the same as those in figure 2. The effect of *K* on the cut-off frequency of the CTHWA is small and only that at G = *1000* is shown in figure 2. Of course, changing *K* will not affect the hot-wire attenuation. Figure 4 shows that the cut-off frequency of the hot-wire system increases from $f_c = 8.2kHz$ at $K = 1000$ to *17 kHz* at $K = 2000$. This shows that in case *5μm* tungsten wire has to be used for turbulence measurements, using high *K* values can increase the cut-off frequency of the hot-wire system. However, increasing *K* will also increase the amplification to the noise within the system such as the electronic noise, and this will in general reduce the signal to noise ratio of the system.

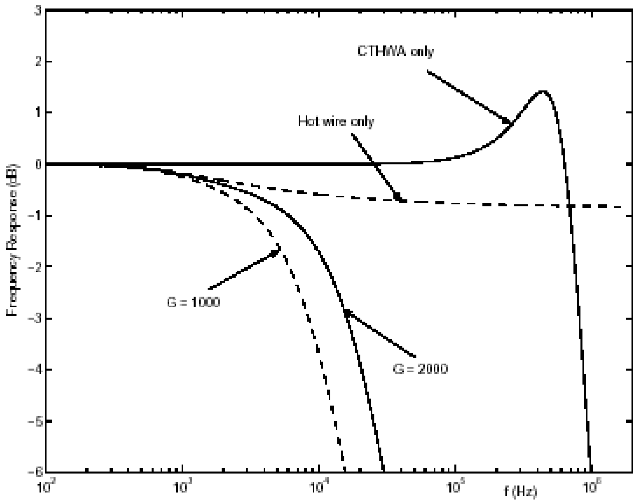

Figure 4 The effect of feedback gain on the cut-off frequency

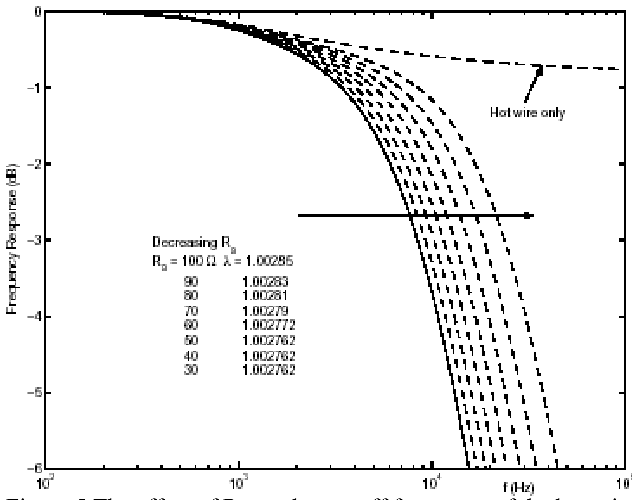

Figure 5 The effect of Ra on the cut-off frequency of the hot-wire system

Figure 5 shows another way to increase the cut-off frequency of the hot-wire system by reducing the resistance R_a. In the figure, *5μm* tungsten wire has been used and the ratio $R_c/R_a = 10$ has been kept to have the bridge close to balance. In order to achieve the optimal frequency response for the CTHWA, λ has also been adjusted and the value is also given in figure 5 for each R_a. Figure 5 shows that the hot-wire system has a cut-off frequency $f_c = 8.2\ kHz$ with $R_a = 100\ \Omega$ and this increases to $f_c = 21\ kHz$ at $R_a = 30\ \Omega$.

Figure 6 shows the effect of L_w on the cut-off frequency of the CTHWA and that of the hot-wire system for a *5μm* tungsten wire. Except the L_w and the λ values, other parameters are the same as those in figure 2. In practice, L_w is determined by the cable length of the hot-wire probe. Perry [8] suggested that for each meter of hot-wire cable there would be an effective *5μH* inductance. In the results presented so far, L_w = *15μH* has been used, which corresponds to a *3m* long hot-wire cable. Figure 6 shows that changing L_w from *5μH* to *30μH* will reduce the cut-off frequency of the CTHWA from *1.5 MHz* to *610 kHz* while there is no appreciable change in the cut-off frequency of the hot-wire system. One interesting point can be noticed from figure 6 is that the overshot peaks increase as L_w increase even though the overshot determined from the zero and pole diagram using MATLAB has always been kept at around 14.7%. This is because the 14.7% overshot determined from the zero and pole plot is based on the overshot at the pole positions and the results shown in figure 6 include the effects from the zeros as well. Nevertheless, it will not affect the conclusions here.

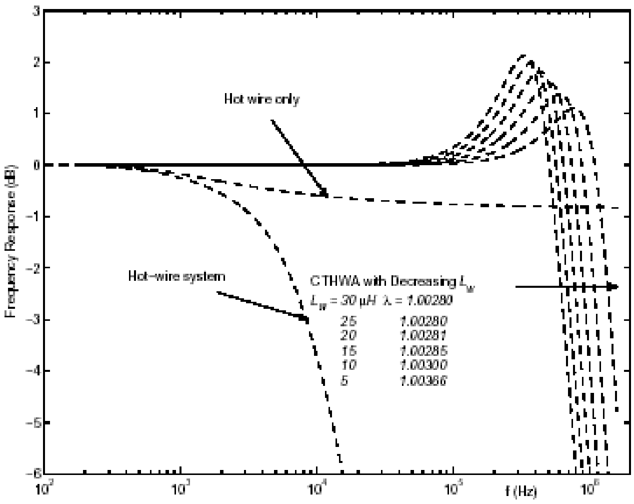

Figure 6 The effect of L_w on the cut-off frequency

The effect of R_b on the cut-off frequency of the hot-wire system has also been investigated. It is found that the cut-off frequency f_c of the CTHWA and the hot-wire system is not affected at all when R_b is changed from R_b = *9.98* R_w to R_b = *.998* R_w (R_c is changed from *1000Ω* to *100Ω* to have the Wheatstone bridge being close to balance, and the overheat ratio has been kept the same).

Conclusions and discussion

The cut-off frequency of the hot-wire system has been investigated by using the theoretical solution of Li [1]. It is found that the hot-wire diameter and the thermal conductivity of the hot-wire material have large effects on the cut-off frequency. The results show that *2.5mm Pt/Ir* wire would have a cut-off frequency in the order of *100 kHz* with a *GBP* > 10^7. This is probably sufficient for most of today's turbulence measurements.

The effect of the *GBP* of the amplifiers used in the feedback circuit is investigated and it is found that the square wave test for optimal frequency response may not guarantee the stability of the hot-wire system. In order to have a stable frequency response for the hot-wire system, *GBP* should be larger than 10^7 for *2.5μm Pt/Ir* wire.

In case *5μm* tungsten wires need to be used for turbulence measurements, it is found that increasing the overall gain *K* in the feedback circuit and using smaller resistance R_a in the Wheatstone bridge can both increase the cut-off frequency of the hot-wire system. However, hot-wire operator should keep in mind that using higher *K* may reduce the signal to noise ratio of the system. It is also found that reducing the resistance R_b or using shorter hot-wire cable (which results in smaller L_w value) will not increase the cut-off frequency of the hot-wire system, although the later can increase the cut-off frequency of the CTHWA.

One point that needs to be mentioned is the attenuation of the hot wire to the velocity fluctuation at high frequencies. The current investigation on the cut-off frequency of the hot-wire system assumes that the frequency response of the system approaches the attenuation curve from below as f_c is increased. Because of this, the measured velocity is that of being attenuated at high frequency. So far, there is no correction method available to take this attenuation effect into account. A calibration method similar to that of [4] is needed to calibrate the hot wire and check its frequency response in free stream flow.

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